

MR3500827 06D22 54B05 54D10

Ball, Richard N. (1-DNV-DM); **Picado, Jorge** (P-CMBR-CM);

Pultr, Aleš (CZ-KARLMP-AM)

On an aspect of scatteredness in the point-free setting. (English summary)

Port. Math. **73** (2016), no. 2, 139–152.

Subfitness and fitness are important separation properties in pointfree topology. (In point-set topology, subfitness is weaker than T_1 -separation, and fitness is similar to, yet weaker than, regularity.) A locale is subfit (resp. fit) if and only if each of its open (resp. closed) sublocales is a join of closed (resp. a meet of open) sublocales. Because, in fact, a locale is fit if and only if *each* of its sublocales is a meet of open sublocales, the natural question then is to characterize the case in which each sublocale of a given locale is a join of closed sublocales. The authors provide a remarkable answer to this question. They prove that this case is equivalent to both subfitness and fitness, with the additional property that the complete lattice of all sublocales of the locale considered is also a locale. The latter property is called scatteredness. *Tomasz Kubiak*

References

1. C. E. Aull and W. J. Thron, Separation axioms between T_0 and T_1 , *Indag. Math.* 24 (1962), 26–37. [MR0138082](#)
2. B. Banaschewski and A. Pultr, Variants of openness, *Appl. Categ. Structures* 2 (1994), 331–350. [MR1300720](#)
3. B. Banaschewski and A. Pultr, Pointfree aspects of the T_D axiom of classical topology, *Quaest. Math.* 33 (2010), 369–385. [MR2755527](#)
4. G. Bruns, Darstellungen und Erweiterungen geordneter Mengen II, *J. Reine Angew. Math.* 210 (1962), 1–23. [MR0143718](#)
5. T. Dube, Submaximality in locales, *Topology Proc.* 29 (2005), 431–444. [MR2244483](#)
6. A. Grothendieck and J. Dieudonné, *Éléments de géométrie algébrique, I: Le langage des schemas*, Grundlehren der Mathematischen Wissenschaften, vol. 166, Springer-Verlag, Berlin, 1971. [MR3075000](#)
7. J. R. Isbell, Atomless parts of spaces, *Math. Scand.* 31 (1972), 5–32. [MR0358725](#)
8. P. T. Johnstone, *Stone Spaces*, Cambridge Univ. Press, Cambridge, 1982. [MR0698074](#)
9. S. Niefield and K. Rosenthal, Spatial sublocales and essential primes, *Topology Appl.* 26 (1987), 263–269. [MR0904472](#)
10. J. Picado and A. Pultr, *Frames and Locales: topology without points*, Frontiers in Mathematics, Vol. 28, Springer, Basel, 2012. [MR2868166](#)
11. J. Picado and A. Pultr, More on subfitness and fitness, *Appl. Categ. Structures* 23 (2015), 323–335. [MR3351084](#)
12. J. Picado and A. Pultr, New aspects of subfitness in frames and spaces, *Appl. Categ. Structures* (to appear). [MR3546508](#)
13. T. Plewe, Higher order dissolutions and Boolean coreflections of locales, *J. Pure Appl. Algebra* 154 (2000), 273–293. [MR1787603](#)
14. T. Plewe, Sublocale lattices, *J. Pure Appl. Algebra* 168 (2002), 309–326. [MR1887161](#)
15. A. Pultr and A. Tozzi, Separation axioms and frame representation of some topological facts, *Appl. Categ. Structures* 2 (1994), 107–118. [MR1283218](#)

16. K. Rosenthal, *Quantales and Their Applications*, Longman, Harlow, 1990. [MR1088258](#)
17. J. Rosický and B. Šmarda, T_1 -locales, *Math. Proc. Cambridge Philos. Soc.* 98 (1985), 81–86. [MR0789721](#)
18. H. Simmons, The lattice theoretic part of topological separation properties, *Proc. Edinburgh Math. Soc. (2)*, 21 (1978), 41–48. [MR0493959](#)
19. H. Simmons, Spaces with Boolean assemblies, *Colloq. Math.* 43 (1980), 23–39. [MR0615967](#)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.