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**New aspects of subfitness in frames and spaces.** (English summary)

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In this article the authors assemble new facts about subfitness and weak subfitness, two of the low separation axioms in topology. Subfitness is weaker than the  $T_1$  axiom and has no relation with  $T_0$ . The  $T_D$  axiom is a low separation axiom that lies strictly between  $T_1$  and  $T_0$ . However, subfitness together with  $T_D$  is equivalent to  $T_1$ . The authors show that in topological spaces, subfitness is strictly weaker than symmetry. They show that a frame  $L$  is  $T_1$ -spatial if and only if  $L$  is  $T_D$ -spatial and subfit, if and only if  $L$  is step-bounded and subfit. Furthermore, a frame  $L$  is a Boolean algebra precisely when it is dually weakly subfit. In addition, the authors define repleteness in spaces and locales and show that a frame  $L$  is subfit if and only if it contains no non-trivial replete sublocale. A space  $X$  is subfit if and only if for every open set  $U$ ,  $U \approx \mathfrak{s}(U) = \bigcup\{U \mid A \text{ closed, } A \subseteq U\}$ , where  $A \approx B$  means that for any open sets  $U, V$ ,  $U \cap A = V \cap A$  if and only if  $U \cap B = V \cap B$ . Finally, another necessary and sufficient condition for subfitness presented by the authors is the validity of the meet formula for the Heyting operation, while a necessary and sufficient condition for weakly subfitness is the validity of the meet formula for pseudocomplementation. *Mack Zakaria Matlabyana*

## References

1. Aull, C.E., Thron, W.J.: Separation axioms between  $t_0$  and  $t_1$ . *Indag. Math.* **24**, 26–37 (1963) [MR0138082](#)
2. Ball, R.N., Picado, J., Pultr, A.: On an aspect of scatteredness in the pointfree setting. *Port. Math.* **73**, 139–152 (2016) [MR3500827](#)
3. Banaschewski, B., Pultr, A.: Variants of openness. *Appl. Categ. Structures* **2**, 331–350 (1994) [MR1300720](#)
4. Banaschewski, B., Pultr, A.: Pointfree aspects of the  $t_D$  axiom of classical topology. *Quaest. Math.* **33**, 369–385 (2010) [MR2755527](#)
5. Dowker, C.H., Strauss, D.:  $T_1$ - and  $T_2$ -axioms for frames. In: *Aspects of Topology*, London Math. Soc. Lecture Note Ser. 93, pp. 325–335. Cambridge University Press, Cambridge (1985) [MR0787838](#)
6. Herrlich, H.: A concept of nearness. *Gen. Topology Appl.* **5**, 191–212 (1974) [MR0350701](#)
7. Herrlich, H., Pultr, A.: Nearness, subfitness and sequential regularity. *Appl. Categ. Structures* **8**, 67–80 (2000) [MR1785838](#)
8. Isbell, J.R.: Atomless parts of spaces. *Math. Scand.* **31**, 5–32 (1972) [MR0358725](#)
9. Johnstone, P.T.: *Stone Spaces*. Cambridge University Press, Cambridge (1982) [MR0698074](#)
10. Klinke, O.: When consistency is redundant, manuscript (2015)
11. Morita, K.: On the simple extension of a space with respect to a uniformity I. *Proc. Japan Acad.* **27**, 65–72 (1951) [MR0048782](#)
12. Picado, J., Pultr, A.: *Frames and Locales: Topology without Points*. *Frontiers in Mathematics*, vol. 28. Springer, Basel (2012) [MR2868166](#)
13. Picado, J., Pultr, A.: (Sub)fit biframes and non-symmetric nearness. *Topology Appl.*

- 168**, 66–81 (2014) [MR3196839](#)
14. Picado, J., Pultr, A.: More on subfitness and fitness. *Appl. Categ. Structures* **23**, 323–335 (2015) [MR3351084](#)
  15. Pultr, A.: Frames. In: Hazewinkel, M. (ed.) *Handbook of Algebra*, vol. 3, pp. 791–858. Elsevier (2003) [MR2035108](#)
  16. Simmons, H.: The lattice theoretic part of topological separation properties. *Proc. Edinburgh Math. Soc.* **21**(2), 41–48 (1978) [MR0493959](#)
  17. Simmons, H.: Regularity, fitness, and the block structure of frames. *Appl. Categ. Structures* **14**, 1–34 (2006) [MR2216640](#)

*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*