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A unified view of the Dedekind completion of pointfree function rings. (English summary)
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Very often, when pointfree results generalize their pointed counterparts, not only is the scope broadened, but the proofs are more penetrating and lucid. Such was the case when the authors constructed the Dedekind completion of the pointfree function ring $C(L)$ in their previous papers [Forum Math. **27** (2015), no. 5, 2551–2585; MR3393371; Algebra Universalis **75** (2016), no. 3, 301–330; MR3515402]. There they presented the construction in three different ways. In the present paper they take another look at the Dedekind completion of $C(L)$, and provide an appropriate unifying framework for the three different descriptions in the earlier papers. The new view is achieved along the following lines. The authors first isolate a certain suitable collection of scales, then construct Galois connections between this collection of scales and each of the three lattices that were used in the earlier constructions of the Dedekind completion. The technique that the authors develop in the present paper enables them to give a much simpler proof of Lemma 3.1 than in their first-stated paper above. The proof in that paper occupies two and half pages and is certainly not for the faint-hearted. The new proof is more elegant, less than half as long, and (unlike the earlier one) does not invoke the ℓ-ring structure of $C(L)$. *Themba Dube*

References

1. B. BANASCHEWSKI, *The real numbers in pointfree topology*, Textos de Matemática, Vol. 12, Universidade de Coimbra, 1997. MR1621835
2. J. GUTIÉRREZ GARCÍA, T. KUBIAK AND J. PICADO, Localic real-valued functions: a general setting, *J. Pure Appl. Algebra* **213** (2009), 1064–1074. MR2498797
3. J. GUTIÉRREZ GARCÍA, I. MOZO CAROLLO AND J. PICADO, Normal semicontinuity and the completion of pointfree function rings, *Algebra Univers.* **75** (2016), 301–330. MR3515402
4. J. GUTIÉRREZ GARCÍA AND J. PICADO, Rings of real functions in pointfree topology, *Topology Appl.* **158** (2011), 2264–2278. MR2838376
5. J. GUTIÉRREZ GARCÍA AND J. PICADO, On the parallel between normality and extremal disconnectedness, *J. Pure Appl. Algebra* **218** (2014), 784–803. MR3149635
6. I. MOZO CAROLLO, *Frame presentations: variants of the reals, rings of functions, their Dedekind completions, and the unit circle*, Ph.D. Thesis, University of the Basque Country, Bilbao, 2015.
7. I. MOZO CAROLLO, J. GUTIÉRREZ GARCÍA AND J. PICADO, On the Dedekind completion of function rings, *Forum Math.* **27** (2015), 2551–2585. MR3393371
8. J. PICADO AND A. PULTR, *Frames and locales: Topology without points*, Frontiers in Mathematics, Vol. 28, Springer, Basel, 2012. MR2868166
9. S.A. STEINBERG, *Lattice-ordered rings and modules*, Springer, New York, 2010. MR2574285

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.