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Gutiérrez García, Javier (E-EHU);
Kubiak, Tomasz [Kubiak, Tomasz[1]] (PL-POZN-MC); **Picado, Jorge** (P-CMBR)

Perfectness in locales. (English summary)

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In topology, a space is called *perfect* if and only if each open set is the union of countably many closed sets or, equivalently, each closed set is the intersection of countably many open sets. The pointfree versions of the above formulations are stated in terms of open sublocales and closed sublocales, and in this paper it is shown that these localic reformulations are not equivalent. Very good comparisons are drawn between the formulations, and a study of perfectness among mildly normal locales is presented. Finally, it is shown that perfectness is invariant under closed, surjective localic maps. *Martin M. Mugochi*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.