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Generating sublocales by subsets and relations: a tangle of adjunctions. (English summary)

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This paper illustrates how the theory of locales provides a richer context for the discussion of the theory of topological spaces.

Given any topological space X and any subspace U of X, the sublocale of the frame $\mathfrak{O}[X]$ of open sets of X which corresponds to the subspace U is associated with the frame congruence

$$\{(P,Q)\in\mathfrak{O}[X]\times\mathfrak{O}[X]: P\cap Y=Q\cap Y\}.$$

However, given any subset $A\subseteq L$ of a frame L, unlike the case for topological spaces where A comes equipped with the subspace topology, A will not be, in general, a sublocale of L. However, the congruence

$$R_A=\{(a,b)\in L\times L:\mathfrak{o}(a)\cap A=\mathfrak{o}(b)\cap A\}$$

produces the sublocale $L/R_A=\mathfrak{sat}(A)$.

The purpose of the paper is to investigate the connection between these sets A and sublocales $\mathfrak{sat}(A)$. Often a clearer picture is obtained by considering a bigger convenient category. In this case the category **SupLat** of sup-lattices (objects are complete lattices and morphisms are functions preserving arbitrary suprema) which contain the category **Frm** of frames and their homomorphisms as a non-full subcategory does the job.

The summary of the main ideas of the paper are as follows:

Sections 2 and 3: In §2 the quotients of sup-lattices are discussed and then specialised to frames. An adjunction between relations on L and subsets of L is arrived at, which is further elaborated in §3.

Sections 4, 5 and 6: When specialising to frames it turns out that $\mathfrak{sat}(A)$ is not always the smallest sublocale containing A. In fact, $\mathfrak{sat}(A)$ may not contain A at all! The situation $A\subseteq\mathfrak{sat}(A)$ is characterised and an illustrative example is provided.

Sections 7 and 8: The adjunction between relations and subsets is further elaborated and in the final section a localic version of the frame quotient theorem is proved.

The paper is nicely written and could be read by anyone who has first been introduced to frames and has an elementary understanding of adjunctions. *Partha Pratim Ghosh*

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