MR3765290 06D22 54C30
Picado, Jorge (P-CMBR-CM); **Pultr, Aleš** (CZ-KARLMP-AM)
A Boolean extension of a frame and a representation of discontinuity. (English summary)
Quaest. Math. **40** (2017), *no. 8,* 1111–1125.

The authors take advantage of their previous work on the frame of closedly generated sublocales in order to model arbitrary real functions, mappings that are not necessarily continuous, in the pointfree setting. Doing so, they mend certain disadvantages of the previous approach, which has otherwise shown to be very successful. Specifically, they replace the frame L by a discretization $\mathsf{S}_{\mathfrak{c}}(L)$ that turns out to be Boolean when L is subfit. Further, when $L = \mathcal{O}X$ for a T_1-space X, $\mathsf{S}_{\mathfrak{c}}(L)$ is isomorphic to the power set of the underlying set of X. In consequence, arbitrary real functions on a T_1-space are in one-to-one correspondence with arbitrary real functions on its frame of open subsets. This new approach does not introduce any obstacles in developing the theory of rings of real functions. Furthermore, the new ring of arbitrary real functions is always Dedekind complete, just like in the classical case. Finally, they show that their approach yields the desired results in the treatment of semicontinuity and regularization of discontinuous functions. *Imanol Mozo Carollo*

References

1. C.E. Aull and W.J. Thron, Separation axioms between T_0 and T_1, *Indag. Math.* **24** (1963), 26–37. MR0138082
2. R.N. Ball, J. Picado and A. Pultr, On an aspect of scatteredness in the point-free setting, *Port. Math.* **73** (2016), 139–152. MR3500827
3. B. Banaschewski, *The real numbers in point-free topology*, Textos de Matemtica, Vol. 12, Universidade de Coimbra, 1997. MR1621835
4. B. Banaschewski and S.S. Hong, Completeness properties of function rings in point-free topology, *Comment. Math. Univ. Carolin.* **44** (2003), 245–259. MR2026162
5. J. Gutirrez Garca, T. Kubiak and J. Picado, Lower and upper regularizations of frame semicontinuous real functions, *Algebra Universalis* **60** (2009), 169–184. MR2491421
6. J. Gutirrez Garca, T. Kubiak and J. Picado, Point-free forms of Dowker's and Michael's insertion theorems, *J. Pure Appl. Algebra* **213** (2009), 98–108. MR2462988
7. J. Gutirrez Garca, T. Kubiak and J. Picado, Localic real functions: A general setting, *J. Pure Appl. Algebra* **213** (2009), 1064–1074. MR2498797
8. J. Gutirrez Garca and J. Picado, On the algebraic representation of semicontinuity, *J. Pure Appl. Algebra* **210** (2007), 299–306. MR2319999
9. J.R. Isbell, Atomless parts of spaces, *Math. Scand.* **31** (1972), 5–32. MR0358725
10. P.T. Johnstone, *Stone Spaces*, Cambridge Univ. Press, Cambridge, 1982. MR0698074
11. Y.-M. Li and G.-J. Wang, Localic Kattov-Tong insertion theorem and localic Tietze extension theorem, *Comment. Math. Univ. Carolinae* **38** (1997), 801–814. MR1603726
12. J. Picado, A new look at localic interpolation theorems, *Topology Appl.* **153** (2006),

3203–3218. MR2259820

13. J. Picado and A. Pultr, *Frames and Locales: topology without points*, Frontiers in Mathematics, Vol. 28, Springer, Basel, 2012. MR2868166

14. J. Picado and A. Pultr, More on subfitness and fitness, *Appl. Categ. Structures* **23** (2015), 323–335. MR3351084

15. J. Picado, A. Pultr and A. Tozzi, Joins of closed sublocales, Pr-Publicaes DMUC 16–39, 2016, (submitted).

16. H. Simmons, The lattice theoretic part of topological separation properties, *Proc. Edinburgh Math. Soc.* **21**(2) (1978), 41–48. MR0493959

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.