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Hedgehog frames and a cardinal extension of normality. **(English summary)**
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We recall from [R. Engelking, *General topology*, translated from the Polish by the author, second edition, Sigma Ser. Pure Math., 6, Heldermann, Berlin, 1989; MR1039321] that for each set I of cardinality κ, the classical metric hedgehog $J(\kappa)$ is the disjoint union $\bigcup_{i\in I}[0,1]\times\{i\}$ of κ copies (the spines) of the real unit interval identified at the origin, with the topology generated by the metric $d\colon J(\kappa)\times J(\kappa)\to[0,+\infty)$ given by

$$d([(t,i)],[(s,j)])=\begin{cases}|t-s| & \text{if } j=i,\\ t+s & \text{if } j\neq i,\end{cases}$$

where $[(t,i)]:=\{(t',i')\in J(\kappa)\colon t=0=t' \text{ or } (t,i)=(t',i')\}$.

In the article under review, the authors generalize a major portion of the theory of the classical metric hedgehog to the setting of pointfree topology as follows. Let I be a set of cardinality κ. The frame of *the metric hedgehog with κ spines* (or simply *the hedgehog frame*) is the frame $\mathfrak{L}(J(\kappa))$ presented by generators $(r,-)_i$ and $(-,r)$ for $r\in\mathbb{Q}$ and $i\in I$, subject to the defining relations:

(h0) $(r,-)_i\wedge(s,-)_j=0$ whenever $i\neq j$;
(h1) $(r,-)_i\wedge(-,s)=0$ whenever $r\geq s$ and $i\in I$;
(h2) $\bigvee_{i\in I}(r_i,-)_i\vee(-,s)=1$ whenever $r_i<s$ for every $i\in I$;
(h3) $(r,-)_i=\bigvee_{s>r}(s,-)_i$ for every $r\in\mathbb{Q}$ and $i\in I$; and
(h4) $(-,r)=\bigvee_{s<r}(-,s)$ for every $r\in\mathbb{Q}$.

They prove that for each cardinal κ, the hedgehog frame $\mathfrak{L}(J(\kappa))$ is a metric frame of weight $\kappa\cdot\aleph_0$, complete in its metric uniformity. Then the authors show that the countable coproduct of the hedgehog frame with κ spines is universal in the class of metric frames of weight $\kappa\cdot\aleph_0$, that is, every metrizable frame of weight $\kappa\cdot\aleph_0$ is embeddable into a countable cartesian power of the hedgehog frame.

A. Pultr [Rend. Circ. Mat. Palermo (2) **1984**, Suppl. No. 6, 247–258; MR0782722] introduced the concept of collectionwise normality in frames, and the authors show that κ-collectionwise normality is hereditary with respect to F_σ-sublocales and invariant under closed maps. Then they present the counterparts of Urysohn's separation and Tietze's extension theorems for continuous hedgehog-valued functions. The paper ends by establishing that $\mathfrak{L}(J(\kappa))\cong\mathfrak{O}(J(\kappa))$ as frames, where the latter is the lattice of open subsets of $J(\kappa)$.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.