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Joins of closed sublocales. **(English summary)**

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Let L be a frame, i.e., a complete lattice in which finite meets distribute over arbitrary joins. The first example of a frame is the topology $\mathcal{O}(X)$ of a topological space X. A frame is called spatial if it is isomorphic to some $\mathcal{O}(X)$. A sublocale of a frame L is the set of all fixed points of a nucleus on L. The paper under review deals with the collection $\mathcal{S}_{\vee^{\mathfrak{c}}}(L)$ of those sublocales of L that are joins of closed sublocales of L. The authors first observe that $\mathcal{S}_{\vee^{\mathfrak{c}}}(L)$ is itself a frame and then consider the question to which extent is $\mathcal{S}_{\vee^{\mathfrak{c}}}(L)$ a coframe or a sublocale of the coframe $\mathcal{S}(L)$ of all sublocales of L. For L a subfit frame (each open sublocale is a join of closed sublocales), the authors give a complete answer. They prove the following:

Theorem 3.5. L is subfit if and only if $\mathcal{S}_{\vee^{\mathfrak{c}}}(L)$ is the Booleanization of $\mathcal{S}(L)$ if and only if $\mathcal{S}_{\vee^{\mathfrak{c}}}(L)$ is a sublocale of $\mathcal{S}(L)$ if and only if $\mathcal{S}_{\vee^{\mathfrak{c}}}(L)$ is a Boolean algebra.

The authors also show, among other things, that when X is T_1, then $\mathcal{S}_{\vee^{\mathfrak{c}}}(\mathcal{O}(X))$ is precisely the set of all $\mathcal{O}(Y)$ with Y a subspace of X. *Tomasz Kubiak*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.