MR3989353 54D35 06D22 54D10
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Some aspects of (non)functoriality of natural discrete covers of locales. (English summary)

Quaest. Math. **42** (2019), *no. 6,* 701–715.

It has been shown in [J. Picado, A. Pultr and A. Tozzi, Houston J. Math. **45** (2019), no. 1, 21–38; MR3951126] that the frame join-generated by closed sublocales of any frame L, denoted by $\mathsf{S}_\mathfrak{c}(L)$, is a frame, and that for subfit frames (and only for them), it is Boolean. Furthermore, as shown in [R. N. Ball and A. Pultr, Algebra Universalis **79** (2018), no. 2, Art. 32; MR3788811], $\mathsf{S}_\mathfrak{c}(L)$ is the maximal essential extension of L for any subfit L.

The authors of the present article show that for any frame L, $\mathsf{S}_\mathfrak{c}(L)$ is an essential extension of L, and that L and $S_\mathfrak{c}(L)$ have the same maximal essential extension, up to isomorphism. The construction $\mathsf{S}_\mathfrak{c}$ is not functorial. Denote by $\mathfrak{o}_L$ the homomorphism, $\mathfrak{o}_L\colon L \to \mathsf{S}_\mathfrak{c}(L)$, that sends an element to the open sublocale it induces. Not every frame homomorphism $h\colon L \to M$ can be "lifted", in the sense of the existence of a homomorphism $\mathsf{S}_\mathfrak{c}(h)\colon \mathsf{S}_\mathfrak{c}(L) \to \mathsf{S}_\mathfrak{c}(M)$ such that $\mathsf{S}_\mathfrak{c}(h) \circ \mathfrak{o}_L = \mathfrak{o}_M \circ h$. The authors thus consider individual liftings, and show that every frame homomorphism with a regular domain and Boolean codomain can be lifted.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.