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Picado, Jorge (P-CMBR-CM); **Pultr, Aleš** (CZ-KARL-AM)
Axiom T_D and the Simmons sublocale theorem. (English summary)
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The paper under review provides a deeper insight into the relationships between subspaces and sublocales of spatial locales (after [H. Simmons, Colloq. Math. **43** (1980), no. 1, 23–39; MR0615967] and [S. B. Niefield and K. I. Rosenthal, Topology Appl. **26** (1987), no. 3, 263–269; MR0904472]).

Let $\Omega(j)\colon \Omega(Y) \to \Omega(X)$ be the frame homomorphism induced by the embedding j of a subspace Y into a topological space X. Let $k\colon \Omega(Y) \to \Omega(X)$ be the localic map adjoint to $\Omega(j)$. The sublocale $S_Y = k(\Omega(Y))$ (= usual image) is said to be *induced* by Y. If the representation $Y \mapsto S_Y$ is bijective (this is the case exactly if X is a T_D-space), then it is called *precise*. Part of the main result of the paper under review (Theorem 3.3) is the following:

Theorem. All sublocales of $\Omega(X)$ are induced and precisely represent subspaces of X if and only if X is a scattered T_D-space.

It should be emphasized that the proof is choice-free. *Tomasz Kubiak*

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