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On hereditary properties of extremally disconnected frames and normal frames. (English summary)

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The notions of normality, complete normality, extremal disconnectedness and complete extremal disconnectedness are lattice-theoretic. The normality properties are dual to the disconnected ones. The authors give several characterisations of completely extremally disconnected frames. They explore the parallelism between completely extremally disconnected frames and completely normal frames. They also consider variants of these concepts with respect to a fixed subset $A \subseteq L$ of a lattice L. One of the main results is the characterisation of when $\mathsf{S}(L)$, the sublocale lattice of L, is completely $\mathcal{A}$-normal, where $\mathcal{A} = \mathfrak{o}[A]$ or $\mathfrak{c}[A]$. On hereditary properties, the main result is that the equivalence of hereditary extremal disconnectedness and complete extremal disconnectedness of the frame of open subsets of a topological space X extends to arbitrary frames. Lastly, they show that in checking hereditary normality of a frame, it is enough to check the property for dense and open sublocales. *Oghenetega Ighedo*

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