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Perfect locales and localic real functions. (English summary)

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In what follows, we assume the reader is familiar with the notions and usual terminology of frames, locales and sublocales. A *localic real function* on a frame L is a frame homomorphism from the frame of reals into the dual of the system of all sublocales of L . A locale L is *normal* if for any $a, b \in L$ with $a \vee b = 1$ there exist $u, v \in L$ such that $a \vee u = 1 = b \vee v$ and $u \wedge v = 0$.

A topological space is *perfect* if each open set is F_σ , and this is equivalent to the statement that each closed set is G_δ . In the category of locales, these two formulations are no longer equivalent (when stated in terms of open sublocales and closed sublocales), the G_δ -perfectness being stronger, in general, than F_σ -perfectness. In their previous work [*Quaest. Math.* **40** (2017), no. 4, 507–518; [MR3665847](#)] the authors took the weaker condition as the pointfree conception of perfectness and kept the terminology of G_δ -perfect locale for the locales which satisfy the stronger condition.

In the paper under review, the authors prove that the strict insertion theorem for real-valued functions on perfect topological spaces extends to both classes of perfect and G_δ -perfect locales (in terms of strict insertion of two comparable lower semicontinuous and upper semicontinuous localic real functions), with a slightly different formulation in the former case. Also, they combine those results with the insertion theorem for normal locales and provide an improved formulation of the pointfree form of Michael's strict insertion theorem for perfectly normal (= normal + G_δ -perfect) locales presented by the authors themselves in [*J. Pure Appl. Algebra* **213** (2009), no. 1, 98–108; [MR2462988](#)].

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.