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Continuous extensions of real functions on arbitrary sublocales and C -, C^* -, and z -embeddings. (English summary)

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Let S be an arbitrary subspace of a topological space X . A theorem of S. Mrówka [Nieuw Arch. Wisk. (3) **16** (1968), 94–111; [MR0244938](#)] states that a bounded continuous function $f: S \rightarrow \mathbb{R}$ has a continuous extension to the whole of X if and only if the sets $[f \leq r]$ and $[f \geq s]$ are completely separated in X for all $r < s$ in $\mathbb{R}$. This theorem has been extended to pointfree topology in [J. Gutiérrez García and T. Kubiak, *J. Pure Appl. Algebra* **215** (2011), no. 6, 1198–1204; [MR2769226](#)] under the assumption that S is a complemented sublocale of a locale L . In the paper under review the authors get rid of the assumption that the sublocale S is complemented. This is the final localic version of Mrówka's theorem. Further results of the paper under review include various characterizations of C -embedding, C^* -embedding and z -embedding of arbitrary sublocales. Normal locales are characterized in these terms. *Tomasz Kubiak*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.