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On equalizers in the category of locales. (English summary)

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In this paper, the authors carefully analyse the connection between equalizers and diagonals in the category **Loc** of locales (which is the dual of the category **Frm** of frames). They exploit the 'covariant' take on locales they developed in [*Frames and locales*, Front. Math., Birkhäuser/Springer Basel AG, Basel, 2012; MR2868166] and [*Separation in point-free topology*, Birkhäuser/Springer, Cham, 2021; MR4241435], making it easier to work with images/pre-images and hence also with pullbacks in **Loc**. For their analysis, they also build upon general categorical facts from [M. M. Clementino, E. Giuli and W. Tholen, in *Categorical foundations*, 103–163, Encyclopedia Math. Appl., 97, Cambridge Univ. Press, Cambridge, 2004; MR2056582]. The authors prove some nice results, e.g. a proposition in 2.6.2, that the locales that appear as domains of monomorphisms in **Loc** with regular codomains form an epireflective subcategory of **Loc**. In Sections 3 and 4, some nice concrete descriptions of equalizers and diagonals are obtained, and they are further refined under the assumption of strong Hausdorffness (or Isbell-Hausdorffness), also providing an alternative of the Dowker-Strauss characterization of strong Hausdorffness (Theorem 4.4.1). In the final section, a characterization of atomic Boolean algebras in terms of open/clopen diagonals is derived (Theorem 5.4.1/Corollary 5.4.2).

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References

1. Adámek, J., Herrlich, H., Strecker, G.E.: Abstract and concrete categories: the joy of cats. Reprints. Theory Appl. Categ. **17**, 1–507 (2006) MR2240597
2. Banaschewski, B.: On pushing out frames. Comment. Math. Univ. Carolin. **31**, 13–21 (1990) MR1056165
3. Clementino, M.M.: Separação e compacidade em categorias. Ph.D. Thesis, Universidade de Coimbra (1991)
4. Clementino, M.M., Giuli, E., Tholen, W.: A functional approach to general topology. In: Categorical Foundations, Encyclopedia Mathematics Applications, vol. 97, pp. 103–163. Cambridge University Press, Cambridge (2004) MR2056582
5. Clementino, M.M., Picado, J., Pultr, A.: The other closure and complete sublocales. Appl. Categ. Struct. **26**, 892–906 (2018). (corr. 907–908) MR3856923
6. Dowker, C.H., Strauss, D.: T_1- *and* T_2-axioms for frames. In: Aspects of Topology, London Mathematical Society. Lecture Note Series, vol. 93, pp. 325–335. Cambridge University Press, Cambridge (1985) MR0787838
7. Hofmann, K.H., Lawson, J.D.: The spectral theory of distributive continuous lattices. Trans. Am. Math. Soc. **246**, 285–310 (1978) MR0515540
8. Isbell, J.R.: Atomless parts of spaces. Math. Scand. **31**, 5–32 (1972) MR0358725
9. Isbell, J.R.: Function spaces and adjoints. Math. Scand. **36**, 317–339 (1975) MR0405340
10. Johnstone, P.T.: Stone Spaces. Cambridge University Press, Cambridge (1982) MR0698074
11. Joyal, A., Tierney, M.: An extension of the Galois Theory of Grothendieck. Mem-

oirs of the American Mathematical Society, vol. 309. AMS, Providence, RI (1984) MR0756176
12. Picado, J., Pultr, A.: Frames and Locales: Topology Without Points, Frontiers in Mathematics, vol. 28. Springer, Basel (2012) MR2868166
13. Simmons, H.: Spaces with Boolean assemblies. Colloq. Math. **43**, 23–29 (1980) MR0615967
14. Townsend, C.: Preframe Techniques in Constructive Locale Theory. Ph.D. Thesis, Department of Computing, Imperial College, London (1996)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.