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Some general aspects of exactness and strong exactness of meets. (English summary)

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Exact meets in a distributive lattice L are the meets $\bigwedge_i a_i$ such that for all b in L ,

$$\left(\bigwedge_i a_i\right) \vee b = \bigwedge_i (a_i \vee b).$$

The concept goes back to H. M. MacNeille [see *Extensions of partially ordered sets*, Ph.D. thesis, Harvard Univ., 1935; [MR2937009](#)] and has been found useful in the study of injective lattices and of Cauchy lattices.

The current paper is structured as follows: 0. Introduction, 1. Preliminaries, 2. φ -Precision and coframe extension of semilattices, 3. ψ -Exactness in particular φ^{op} -exactness and exactness, 4. The right hand sides.

From the summary: “strongly exact meets in a frame are preserved by all frame homomorphisms. Finite meets are, trivially, (strongly) exact; this naturally leads to the concepts of exact resp. strongly exact filters closed under all exact resp. strongly exact meets. In [R. N. Ball, M. A. Moshier and A. Pultr, *Appl. Categ. Structures* **28** (2020), no. 4, 655–667; [MR4114995](#); M. A. Moshier, A. Pultr and A. L. Suarez, *Appl. Categ. Structures* **28** (2020), no. 6, 907–920; [MR4163308](#)] it was shown that the subsets of all exact resp. strongly exact filters are sublocales of the frame of up-sets on a frame, hence frames themselves, and, somewhat surprisingly, that they are isomorphic to the useful frame $S_c(L)$ of sublocales join-generated by closed sublocales resp. the dual of the coframe meet-generated by open sublocales.

“In this paper we show that these are special instances of much more general facts. The latter concerns the free extension of join-semilattices to coframes; each coframe homomorphism lifting a general join-homomorphism $\varphi: S \rightarrow C$ (where S is a join-semilattice and C a coframe) and the associated (adjoint) colocalic maps create a frame of generalized strongly exact filters (φ -precise filters) and a closure operator on C (and – a minor point – any closure operator on C is thus obtained). The former case is slightly more involved: here we have an extension of the concept of exactness (ψ -exactness) connected with the lifts of $\psi: S \rightarrow C$ with complemented values in more general distributive complete lattices C creating, again, frames of ψ -exact filters; as an application we learn that if such a C is join-generated (resp. meet-generated) by its complemented elements then it is a frame (resp. coframe) explaining, e.g., the coframe character of the lattice of sublocales, and the (seemingly paradoxical) embedding of the frame $S_c(L)$ into it.”

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References

1. R.N. Ball, Distributive Cauchy lattices, *Algebra Universalis* **18** (1984) 134–174. [MR0743464](#)
2. R.N. Ball, M.A. Moshier, A. Pultr, Exact filters and joins of closed sublocales, *Appl. Categ. Structures* **28** (2020) 655–667. [MR4114995](#)

3. R.N. Ball, J. Picado, A. Pultr, Notes on exact meets and joins, *Appl. Categ. Structures* 22 (2014) 699–714. [MR3275269](#)
4. R.N. Ball, J. Picado, A. Pultr, On an aspect of scatteredness in the point-free setting, *Port. Math. (N.S.)* 73 (2016) 139–152. [MR3500827](#)
5. R.N. Ball, J. Picado, A. Pultr, Some aspects of (non)functoriality of natural discrete covers of locales, *Quaest. Math.* 42 (2019) 701–715. [MR3989353](#)
6. G. Bruns, H. Lakser, Injective hulls of semilattices, *Canad. Math. Bull.* 13 (1970) 115–118. [MR0274372](#)
7. M.M. Clementino, J. Picado, A. Pultr, The other closure and complete sublocales, *Appl. Categ. Structures* 26 (2018) 892–906. [MR3856922](#)
8. J.R. Isbell, Atomless parts of spaces, *Math. Scand.* 31 (1972) 5–32. [MR0358725](#)
9. P.T. Johnstone, *Stone Spaces*, Cambridge Studies in Advances Math., vol. 3, Cambridge University Press, 1982. [MR0698074](#)
10. H.M. MacNeille, Extensions of partially ordered sets, *Proc. Natl. Acad. Sci. USA* 22 (1936) 45–50. [MR2937009](#)
11. J. Martínez, W. McGovern, Free meets and atomic assemblies of frames, *Algebra Universalis* 62 (2009) 153–163. [MR2661372](#)
12. M.A. Moshier, A. Pultr, A.L. Suarez, Exact and strongly exact filters, *Appl. Categ. Structures* 28 (2020) 907–920. [MR4163308](#)
13. J. Picado, A. Pultr, *Frames and Locales: Topology without Points*, *Frontiers in Mathematics*, vol. 28, Springer, Basel, 2012. [MR2868166](#)
14. J. Picado, A. Pultr, A. Tozzi, Joins of closed sublocales, *Houston J. Math.* 45 (2019) 21–38. [MR3951126](#)
15. J. Todd Wilson, *The assembly tower and some categorical and algebraic aspects of frame theory*, PhD Thesis, Carnegie Mellon University, 1994.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.