MR4574408 54D15 06D22 18F70

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Frame presentations of compact hedgehogs and their properties. (English. English summary)

Quaest. Math. **46** (2023), *no.* 2, 207–242.

A point-free form of the usual metric hedgehog with κ spines has been investigated by J. Gutiérrez García et al. [J. Pure Appl. Algebra **223** (2019), no. 6, 2345–2370; MR3906552]. In the literature, there is also the so-called compact hedgehog with κ spines which can be viewed as the subspace of the Tikhonov cube $[0,1]^\kappa$ consisting of its "coordinate axes".

In the paper under review, the authors introduce a point-free form of the compact hedgehog, denoted $\mathcal{L}(cJ(\kappa))$, via a clever frame presentation by generators and relations. Given a frame L and the coframe $\mathcal{S}(L)$ of all sublocales of L, the paper introduces several forms of frame homomorphisms from $\mathcal{L}(cJ(\kappa))$ into $\mathcal{S}(L)^{op}$, including continuous, lower semicontinuous and upper semicontinuous ones. Using these homomorphisms, the authors provide various insertion and extension theorems (including disjoint extension, the idea of which goes back to M. Frantz [Pacific J. Math. **169** (1995), no. 1, 53–73; MR1346246]).

In the first half of the paper, the authors characterize κ-collectionwise normal frames in terms of the κ-disjoint extension property, as well as normal frames in terms of the Katětov-Tong type insertion theorem, thereby giving point-free versions of results of Gutiérrez García, T. Kubiak and M. A. de Prada-Vicente [Houston J. Math. **35** (2009), no. 2, 469–484; MR2519542]. In the second half of the paper, the authors deal with a common setting for both normality and total κ-collectionwise normality. Several other variants of *normality* are thus covered, including mild normality and extremal disconnectedness, among others. To achieve this, the authors use general ideas developed in [J. Gutiérrez García and J. Picado, J. Pure Appl. Algebra **218** (2014), no. 5, 784–803; MR3149635]. Roughly speaking, some insertion and extension theorems concerning frame homomorphism from $\mathcal{L}(\mathbb{R})$ (= the frame of reals) into $\mathcal{S}(L)^{op}$ proved in the paper just cited are shown to hold true in the context of $\mathcal{L}(cJ(\kappa))$. *Tomasz Kubiak*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.