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Gutiérrez García, Javier (E-BILB); **Kubiak, Tomasz [Kubiak, Tomasz[1]]** (PL-POZN-MC); **Picado, Jorge** (P-CMBR-CM)
Localic real functions: a general setting. (English summary)
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Given a topological space X, it is sometimes desirable to consider real-valued functions on X which are not necessarily continuous. For instance, a topological space X is said to be a Bluemberg space if for every (not necessarily continuous) function $f\colon X \to \mathbb{R}$ there is a dense subspace D of X such that the restriction $f|_D\colon D \to \mathbb{R}$ is continuous. In pointfree topology the only real-valued functions on a frame L that have hitherto been considered are the continuous and the semicontinuous ones. In this paper the authors (continuing their programme of extending to the pointfree context real-valued functions [see Algebra Universalis **60** (2009), no. 2, 169–184; MR2491421 (2010e:54015); J. Pure Appl. Algebra **213** (2009), no. 6, 1064–1074; MR2498797; J. Pure Appl. Algebra **213** (2009), no. 1, 98–108; MR2462988]) define a real-valued function on a frame as follows. Starting with a frame L, they take an isomorphic copy of the assembly of L in the form of the lattice of sublocales of L turned upside down and denote it by $\mathcal{S}(L)$. Then, with $\mathfrak{L}(\mathbb{R})$ denoting the frame of reals, they regard the lattice-ordered ring $\mathrm{Frm}(\mathfrak{L}(\mathbb{R}), \mathcal{S}(L))$ as the frame analogue of the ring of all real-valued functions on a topological space. The continuous (respectively, lower and upper semicontinuous) functions in $\mathrm{Frm}(\mathfrak{L}(\mathbb{R}), \mathcal{S}(L))$ are defined akin to topology; to wit, a function F in $\mathrm{Frm}(\mathfrak{L}(\mathbb{R}), \mathcal{S}(L))$ is continuous if $F(p,q)$ is a closed sublocale for all $(p,q) \in \mathfrak{L}(\mathbb{R})$. Semicontinuous functions are defined similarly. Appended to the end of the paper are much more lucid formulations of the authors' results from their papers cited above, expressed in terms of ideas developed in the current paper.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.