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Lower and upper regularizations of frame semicontinuous real functions.
(English)

The main gist of this paper is a characterization of extremally disconnected frames (also known as De Morgan frames) in terms of extendibility of bounded real-valued continuous functions on open sublocales. The main theorem states that a frame L is extremally disconnected if and only if, for every open sublocale S, every bounded real-valued continuous function on S extends to a bounded real-valued continuous function on L. The authors establish this result by first defining the pointfree concepts of lower and upper regularizations of frame semicontinuous real-valued functions.

The stated theorem is a much strengthened version of an earlier effort in this regard by *Y.-M. Li* and *Z.-H. Li* ["Constructive insertion theorems and extension theorems on extremally disconnected frames", Algebra Univers. 44, No. 3–4, 271–281 (2000; Zbl 1013.06010)], which was demonstrated by *J. Picado* ["A new look at localic interpolation theorems", Topology Appl. 153, No. 16, 3203–3218 (2006; Zbl 1104.06007)] to have shortcomings emanating from a treatment of pointfree semicontinuity which was not fully reflective of its point-sensitive counterpart. The preparatory results in the present paper are a continuation of the authors' study of semicontinuity in pointfree topology.

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 54G05 Extremally disconnected spaces, etc.