**Zbl 1154.06006**

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**Pointfree forms of Dowker's and Michael's insertion theorems.** (English)

J. Pure Appl. Algebra 213, No. 1, 98-108 (2009). ISSN 0022-4049
http://dx.doi.org/10.1016/j.jpaa.2008.05.010
http://www.sciencedirect.com/science/journal/00224049

The authors prove two strict insertion theorems for frame homomorphisms.
When applied to the frame of all open subsets of a topological space, they are equivalent to the following insertion theorems of Dowker and Michael regarding, respectively, normal countably paracompact spaces and perfectly normal spaces.
(Dowker) A topological space $X$ is normal and countably paracompact if and only if, given $h, g : X \to \mathbb{R}$ such that $h < g, h$ is upper semicontinuous and $g$ is lower semicontinuous, there is a continuous $f : X \to \mathbb{R}$ such that $h < f < g$.
(Michael) A topological space $X$ is perfectly normal if and only if, given $h, g : X \to \mathbb{R}$ such that $h \leq g, h$ is upper semicontinuous and $g$ is lower semicontinuous, there is a continuous $f : X \to \mathbb{R}$ such that $h \leq f \leq g$ and $h(x) < f(x) < g(x)$ whenever $h(x) < g(x)$.
The authors state that, however, in the pointfree context they have been unable to provide pointfree assertions corresponding exactly to these two classical insertion results.
Furthermore, they present a study of the concept of perfect normality for frames.

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*Keywords* : perfectly normal; insertion theorem; pointfree topology; frame; semicontinuous; normal; countably paracompact
*Classification* :
- ∗06D22 Frames etc.
- 26A15 Continuity and related questions (one real variable)
- 54C30 Real-valued functions on topological spaces
- 54D15 Higher separation axioms