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Completely normal frames and real-valued functions. (English)

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In an earlier publication [J. Pure Appl. Algebra 213, No. 6, 1064–1074 (2009; Zbl 1187.06005)] *J. Gutiérrez García, T. Kubiak* and *J. Picado* define the pointfree version of real-valued (not necessarily continuous or semicontinuous) functions. In the paper under review, the authors use this notion to characterize completely normal locales. Given a locale L, take its sublocale lattice and put it on its head, so that you have a frame $\mathcal{S}(L)$ instead of a co-frame. Dually to the point-sensitive case, two sublocales of L are said to be separated if each is co-disjoint (in the frame $\mathcal{S}(L)$) from the closure of the other. Further, two sublocales S and T of L are said to be separated by open sublocales if there are co-disjoint open sublocales U and V such that $S \geq U$ and $T \geq V$. The authors then define a locale to be completely normal in case every pair of separated sublocales is separated by open sublocales.

As expected, a locale is called hereditarily normal if every sublocale is normal. The authors give numerous characterizations of complete normality in terms of elements and sublocales, including the following:

A locale is completely normal iff it is hereditarily normal iff every open sublocale is normal.

After recalling the notions of lower and upper regularization of a real-valued function on a locale, which were introduced in [*J. Gutiérrez García, T. Kubiak* and *J. Picado*, Algebra Univers. 60, No. 2, 169–184 (2009; Zbl 1181.06003)], the authors proceed to characterize completely normal locales as those with the property that if two real-valued functions on them are comparable (and satisfy some other feature related to upper and lower regularizations), then a lower semicontinuous function and its upper regularization can be inserted between the two functions.

The paper contains many other interesting results, and is written in the authors' inimitable style that makes it easy to read. The authors promise that some ramifications of the current paper will be the subject of a forthcoming paper – something certainly to look forward to.

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Keywords : localic real function; completely normal frame; completely normal space; extremally disconnected frame; insertion; upper and lower regularization; completely normal locales; sublocale lattice

Classification :

∗06D22 Frames etc.
54C30 Real-valued functions on topological spaces
54D15 Higher separation axioms
54G05 Extremally disconnected spaces, etc.