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★ **Categorical foundations.**

Special topics in order, topology, algebra, and sheaf theory.

Edited by Maria Cristina Pedicchio and Walter Tholen.

Encyclopedia of Mathematics and its Applications, 97.

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Contents: R. J. Wood [Richard James Wood], Ordered sets via adjunctions (5–47) [MR2056580](#); Jorge Picado, Aleš Pultr and Anna Tozzi, Locales (49–101) [MR2056581](#); Maria Manuel Clementino, Eraldo Giuli and Walter Tholen, A functional approach to general topology (103–163) [MR2056582](#); Dominique Bourn and Marino Gran, Regular, protomodular, and abelian categories (165–211) [MR2056583](#); John MacDonald and Manuela Sobral [Maria Manuela Sobral], Aspects of monads (213–268) [MR2056584](#); Maria Cristina Pedicchio and Fabrizio Rovatti, Algebraic categories (269–309) [MR2056585](#); Claudia Centazzo and Enrico M. Vitale, Sheaf theory (311–357) [MR2056586](#); George Janelidze, Manuela Sobral [Maria Manuela Sobral] and Walter Tholen, Beyond Barr exactness: effective descent morphisms (359–405) [MR2056587](#).

The book under review consists of eight chapters providing introductions to, among other things, orders, algebraic theories, topology, and the theory of sheaves. The largely independent presentations share the common attitude that the language of category theory can be fruitfully employed to study structures in quite diverse areas of mathematics. Only modest knowledge of category theory is needed to delve into the different topics.

Chapter I deals with ordered sets, partially ordered sets, (distributive) lattices, Heyting algebras, and completely distributive lattices. Constructive reasoning is employed throughout, and the results are valid in any (elementary) topos. However, the reader may first notice that constructive reasoning is employed throughout once the axiom of choice is shown to be equivalent to complete distributivity of power sets, and classical (Boolean) reasoning is shown to be equivalent to the equivalence of different formulations of complete distributivity.

Chapter II is about locales, where the focus is on the algebraic structure of open sets of topological spaces, and sup-complete lattices where finite meets distribute over infinite suprema. The relationship between the category of topological spaces and the category of locales is treated. Closure properties of the category of locales are covered, as well as open, closed, and stably closed (proper) maps. The last sections are devoted to compact and locally compact locales.

Chapter III studies categories equipped with a factorization system that interacts with a class of morphisms resembling closed (continuous) maps. Examples of categories with such a structure are, besides the category of topological spaces with closed maps, locales with closed maps, toposes with closed maps with respect to a closure operator, algebras with Zariski-closed maps, topological spaces with open maps, abelian groups with torsion-preserving maps, etc. In the abstract setting, various notions like proper maps, perfect maps, Hausdorff and Tikhonov spaces, Stone-Čech compactification, exponential maps and spaces are studied, including their meaning in the various examples.

Chapter IV gives an introduction to protomodular categories, categories in which the Short Five Lemma of homological algebra is part of the axiomatization. When further exactness properties are imposed, objects in protomodular categories behave like groups.

Chapter V is devoted to aspects of the theory of monads, including various monadicity theorems and monads in the setting of 2-categories.

Chapter VI on algebraic categories is a primer on categorical logic, albeit in the simplest case of algebraic theories, theories axiomatized by identities in a first-order language consisting of constants and function symbols only. The categorical treatment goes back to F. W. Lawvere's Ph.D. thesis in 1963 ["Functional semantics of algebraic theories", Columbia Univ., New York], and the chapter presents Lawvere's characterization of categories of models of a one-sorted algebraic theory as exact categories with a regular generator that is regular projective and finitely presentable.

The last section deals with locally finitely presentable categories, categories equivalent to categories of finite limit preserving set-valued functors with domain a finitely complete category. Thus, these are categories of models for theories slightly stronger than algebraic.

Chapter VII is a primer to sheaf theory, covering both sheaves over a topological space (equivalently, étale bundles over a space), and the theory of sheaves over a site in the sense of Grothendieck. The chapter ends with Giraud's characterization of Grothendieck toposes as exact, extensive categories with a small generator.

Chapter VIII has as topic (effective) descent morphisms, which loosely speaking are morphisms that allow one to describe problems and structure over a complicated co-domain in terms of structure over a simpler domain. The presentation ends with applications treating extensions in homological algebra, separable field extensions, and locally trivial fibre bundles and covering spaces in algebraic topology.

Reviewed by *Carsten Butz*

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