

Picado, Jorge; Pultr, Aleš

Notes on the product of locales. (English) Zbl 06443684

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In the paper under review the authors describe the product of locales (coproducts of frames) from a number of different perspectives, based on well-known constructions in various categories, with the aim to make the process natural and geometrically transparent. Indeed a product of locales may not be very intuitive as well as its connection with the product of spaces may not be very transparent. Basically this is due to the fact that the category of locales is bigger than that of topological spaces and, as the dual of the category of frames, is a category of algebraic objects. Concretely they first analyse the algebraic aspects by showing a strict similarity of the construction of the product of locales with that of the tensor product of abelian groups: frames enrich a certain structure in a distributive manner in full analogy with that of the multiplication of commutative rings enriching the abelian group structure. Then they describe what happens in a product of locales from the geometric (topological) point of view: in perfect analogy with topological spaces, where the open sets are unions of rectangles $U \times V$ (U, V open), here they are “sums of rectangles” $a \oplus b$. The authors show that the difference between $X \times Y$ in the category of spaces and in the category of locales is in that the sums may be somewhat looser, and explain when they are exactly the same. Furthermore, in the last section, they show how in some cases (paracompact locales, uniform locales) this difference can in fact be beneficent rather than an unpleasant consequence of the generalization.

Reviewer: Anna Tozzi (L'Aquila)

MSC:

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