Mozo Carollo, Imanol; **Gutiérrez García, Javier**; **Picado, Jorge**
On the Dedekind completion of function rings. (English) ｜Zbl 06479260｜
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The frame of reals, $\mathfrak{L}(\mathbb{R})$, is generated by pairs (p, q) of all rationals subject to certain relations that include the relation (R2) which states that $(p, q) \vee (r, s) = (p, s)$ whenever $p \leq r < q \leq s$. The ℓ-ring of real continuous functions on a frame L consists of frame homomorphisms $\mathfrak{L}(\mathbb{R}) \to L$, and is in this paper (and elsewhere) denoted by C(L). The axiom (R2) is, in general, an obstacle to the completeness (as an ℓ-ring) of CL. Indeed, this ℓ-ring is order complete precisely when L is extremally disconnected [*B. Banaschewski* and *S. S. Hong*, Commentat. Math. Univ. Carol. 44, No. 2, 245–259 (2003; Zbl 1098.06006)]. In order to obviate the obstacle (R2), the authors of the present paper delete it from the list of relations required for $\mathfrak{L}(\mathbb{R})$. They thus construct a new device, denoted $\mathfrak{L}(\mathbb{IR})$, that they call the frame of partially defined real numbers. It is a frame presented by the same generators as $\mathfrak{L}(\mathbb{R})$, and by all relations except (R2). They then use this device to construct the order completion of C(L). The construction goes in various steps, each of which involves some heavy nontrivial calculations in the frame $\mathfrak{L}(\mathbb{IR})$. The authors also investigate the cases of bounded real functions as well as integer-valued functions. Often pointfree topology techniques enrich classical topology not only by extending its breadth, but also by providing more lucid and transparent reasons for why things happen the way they do. This paper bears further testimony to that. As an application of their tools, the authors give a new construction of the Dedekind completion of the ℓ-ring C(X) that is, to echo their words, "more direct and natural than the known procedure using Hausdorff continuous functions".

Reviewer: Themba Dube (Unisa)

MSC:

		Cited in **1** Document
06D22	Frames, locales	
06B23	Complete lattices, completions	
54C30	Real-valued functions on topological spaces	
26A15	Continuity and related questions (one real variable)	

Keywords:

frame of reals; frame of partial reals; continuous real functions; partial real function; order complete; Dedekind completion; completely regular frame; extremally disconnected frame; zero-dimensional frame

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