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Presenting the frame of the unit circle. (English) Zbl 06506971

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The frame of reals, $\mathfrak{L}(\mathbb{R})$, can be presented as a frame generated by pairs (p, q) of all rationals subject to some relations. It can also be presented as a frame generated by abstract “rays” $(p, -)$ and $(-, q)$, for $p, q \in \mathbb{Q}$, subject to relations that mimic, in a way, how the rays (p, ∞) , $(-\infty, p)$ relate. Since $\mathfrak{L}(\mathbb{R})$ is isomorphic to the frame of open sets of the line, it is natural to ask if the frame of open sets of the unit circle can be presented by generators and relations. Incidentally, this question was posed to the authors by Bernhard Banaschewski. In answering this, the authors present frame $\mathfrak{L}(\mathbb{T})$ of the unit circle by generators and relations in two alternative ways. The first is the point-free counterpart of the Alexandroff compactification of the real line; and the second can be viewed as a localic analogue of the quotient space \mathbb{R}/\mathbb{Z} . They prove, among other things, that the spectrum of $\mathfrak{L}(\mathbb{T})$ is homeomorphic to the unit circle; which accords with the case of the frame of reals. They show how the usual group operations of $\mathfrak{L}(\mathbb{R})$ can be lifted to $\mathfrak{L}(\mathbb{T})$, thus endowing it with a canonical localic group structure. As they remark, this is forward looking to a prospective point-free description of Pontryagin duality.

Reviewer: **Themba Dube (Unisa)**

MSC:

- 06D22 Frames, locales
- 18D35 Structured objects in a category
- 22A99 Topological algebraic systems
- 54H11 Topological groups (topological aspects)

Keywords:

frame; frame of reals; frame of the unit circle; localic group

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