

Ferreira, Maria João; Matlabyana, Mack; Picado, Jorge

On permutable pairs of quasi-uniformities. (English) Zbl 06509449
Topology Appl. 196, 260-273 (2015).

The paper deals with two important properties of a topological space namely those of being normal and extremally disconnected, respectively. These two features of a space are dual to each other in some sense which was firstly observed in lattice-theoretical terms.

But this duality is not completely symmetric in the sense that not every result in each pair is directly obtainable from its dual one. Recently, *E. P. de Jager* and *H.-P. A. Künzi* [Topology Appl. 158, No. 7, 930–938 (2011; Zbl 1215.54010)] proved a result in the realm of a Pervin quasi-uniformity \mathcal{P} on a topological space X as follows:

- (1) $\mathcal{P} \circ \mathcal{P}^{-\infty}$ is a quasi-uniformity iff X is normal;
- (2) $\mathcal{P}^{-\infty} \circ \mathcal{P}$ is quasi-uniformity iff X is extremally disconnected;
- (3) \mathcal{P} and $\mathcal{P}^{-\infty}$ permute iff X is normal and extremally disconnected.

The primary goal of the present paper is to investigate whether it is possible to formulate Theorem (1) in such a manner that the proof of assertion (2) and (3) is a direct consequence of (1) by some kind of dualization process. Additionally, the extending setting should give the possibility for the formulation and unification of several weak variants of normality.

To manage the above mentioned approach the authors use the idea of selecting different classes of subspaces of the underlying space of the quasi-uniform space.

Consequently this established, they can deal with relative notions of normality and extremally disconnectedness, unifying the different variants.

Moreover, this paper is written in such a way that definitions and obtaining results can be easily extended to the point-free settings of frames and locales.

Reviewer: Dieter Leseberg (Berlin)

MSC:

- 54E15 Uniform structures and generalizations
- 54D15 Higher separation axioms
- 54E55 Bitopologies
- 54G05 Extremally disconnected spaces, F -spaces, etc.
- 54H12 Topological lattices (topological aspects)

Keywords:

entourage; quasi-uniformity; quasi-uniform space; lattice of quasi-uniformities; permutable quasi-uniformities; Pervin quasi-uniformity; normal space; extremally disconnected space; weak variants of normality; pairs of quasi-uniformities

Full Text: DOI

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