Ball, Richard N.; **Picado, Jorge**; **Pultr, Aleš**
On an aspect of scatteredness in the point-free setting. (English) ‖Zbl 06580417‖
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In his pioneering paper [Math. Scand. 31, 5–32 (1972; Zbl 0246.54028)], *J. R. Isbell* introduced the localic separation axioms of fitness and subfitness. These axioms have recently been studied further by other authors, including the second and third authors of the paper under review [Appl. Categ. Struct. 23, No. 3, 323–335 (2015; Zbl 06447447)]. A locale is fit if and only if each of its closed sublocales is a meet of open ones, and subfit if and only if each of its open sublocales is a join of closed sublocales. Since there is the characterisation that a locale is fit if and only if each of its sublocales is a meet of closed ones, it is natural to ask which locales have the property that every sublocale is a join of closed sublocales. It is this question that the authors address. They prove that the locales with this feature are precisely the ones that are simultaneously scattered (meaning that every sublocale is complemented) and subfit or fit. Applied to spatial locales, they show that the frame of open sets of a topological space has this feature if and only if the space is a sober scattered T_1-space. One of the interesting results they prove en passant is that in a regular frame the pseudocomplement of any element is the meet of those elements it joins at the top. It has since been shown by the second and third authors of this paper that this result holds already for subfit frames.

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MSC:

06D22   Frames, locales
54B05   Subspaces (general topology)
54D10   Lower separation axioms (T_0–T_3, etc.)

Keywords:

frame; locale; sublocale; open sublocale; closed sublocale; fit; subfit; scattered

Full Text: DOI