There exists the concept of adjoint pair of maps between partially ordered sets (posets), i.e., given posets $A$, $B$, and maps $f : A \to B$, $g : B \to A$, if $f(a) \leq b$ if and only if $a \leq g(b)$ for every $a \in A$ and every $b \in B$, then $g$ is said to be the right or upper adjoint of $f$, and $f$ is said to be the left or lower adjoint of $g$. An example of an adjoint pair is provided by any map $f : S \to T$ between sets, namely, $f$ induces an adjoint pair of maps $f_\rightarrow : \mathcal{P}S \to \mathcal{PT}$ and $f^{\leftarrow} : \mathcal{PT} \to \mathcal{PS}$ between the powersets $\mathcal{PS}$ and $\mathcal{PT}$ defined by $f_\rightarrow(U) = \{f(u) \mid u \in U\}$ for every $U \in \mathcal{PS}$ and $f^{\leftarrow}(V) = \{s \in S \mid f(s) \in V\}$ for every $V \in \mathcal{PT}$.

The above example could be extended to two functors between the category Top of topological spaces and continuous maps, and the categories Frm of frames and frame homomorphisms (maps preserving arbitrary joins and finite meets), and Loc of locales (another word for frames) and locale morphisms or localic maps (right adjoints of frame homomorphisms), which are the cornerstones of point-free topology [P. T. Johnstone, Stone spaces. Cambridge: Cambridge University Press (1982; Zbl 0499.54001)]. More precisely, there exists a contravariant functor $O^{\leftarrow} : \text{Top} \to \text{Frm}$ defined on a continuous map $f : S \to T$ by $O^{\leftarrow} f : OT \to OS$, $O^{\leftarrow} f(V) = f^{\leftarrow}(V)$, where $OS$ and $OT$ are the topologies on the spaces $S$ and $T$, respectively. Moreover, there exists a functor $O_\rightarrow : \text{Top} \to \text{Loc}$ defined on a continuous map $f : S \to T$ by $O_\rightarrow f : OS \to OT$, $O_\rightarrow f(U) = T \setminus \text{cl}(f_\rightarrow(S \setminus U))$, where $\text{cl}(\cdot)$ stands for the topological closure.

The present paper considers localic maps as “continuous maps” and characterizes them by certain concrete continuity properties in the closure-theoretical sense (preimages of closed subobjects are closed, and the formation of preimages commutes with supplementation). To be more general, the authors study the category of implicial semilattices, i.e., meet-semilattices with top elements, where the unary meet operations $\lambda_a = a \land -$ have right adjoints $\alpha_a = a \to -$. One should observe that frames and locales are just the complete implicial semilattices. In order to have Frm and Loc as full subcategories of two respective dual categories, whose objects are implicial semilattices, the authors consider as morphisms not the usual implicial homomorphisms (preserving finite meets and the binary operation $\to$), but the left adjoint top-preserving semilattice homomorphisms, briefly referred to as r-morphisms, and in the opposite direction their right adjoints, the so-called localizations [G. Bezhanishvili and S. Ghilardi, Ann. Pure Appl. Logic 147, No. 1–2, 84–100 (2007; Zbl 1123.03055)] or l-morphisms. In one word, r-morphisms have right adjoints and preserve finite meets, whereas l-morphisms have left adjoints that preserve finite meets.

The paper is well written, gives most of its required preliminaries (the omitted concepts can be found in one of the references at the end of the paper), and will be of interest to the researchers studying point-free topology.

Reviewer: Sergejs Solovjovs (Praha)
Full Text: DOI

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