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A new diagonal separation and its relations with the Hausdorff property. (English)

Zbl 07502748

Appl. Categ. Struct. 30, No. 2, 247-263 (2022)

From the facts that the Hausdorff property of a topological space X is characterized by the closedness of the diagonal in $X \times X$ and the strong Hausdorff axiom in the category of locales (introduced by *J. R. Isbell* [Math. Scand. 31, 5–32 (1972; Zbl 0246.54028)]), the authors introduce the following interesting and useful criterion for separation: “Let $\mathcal{P}$ be a property of subobjects relevant in a category $\mathcal{C}$. An object $X \in \mathcal{C}$ is $\mathcal{P}$ -separated if the diagonal in $X \times X$ has $\mathcal{P}$ ”.

Using this idea, the authors study the locales (frames) in which the diagonal is fitted (i.e., an intersection of open sublocales – they speak about $\mathcal{F}$ -separated locales). Since the intersection of open sublocales is an operation of closure type, it is natural to ask about fitted diagonals; i.e, the property $\mathcal{F}\mathbf{sep}$ or of the $\mathcal{F}$ -separated locales (frames). The study of this property is the main topic of this paper.

On the other hand, taking into account that a well-known property of a locale (frame) is fitness [*J. R. Isbell*, Math. Scand. 31, 5–32 (1972; Zbl 0246.54028)], that a locale is *fit* if each of its sublocales is fitted, and that fitness is preserved under products and sublocales, the authors make an immediate observation that *fit* implies $\mathcal{F}\mathbf{sep}$. Hence the first question one may ask is whether this implication can be reversed (it cannot; $\mathcal{F}\mathbf{sep}$ is strictly weaker than fitness, which is one of the results of this paper).

Later, the authors prove the Dowker-Strauss type characterization of the strong Hausdorff property, and Finally, they prove that $\mathcal{F}\mathbf{sep}$ is strictly weaker than fitness, and that it does not coincide with any of the three standard axioms weaker than fitness: subfitness, weak subfitness and prefitness.

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MSC:

18F70 Frames and locales, pointfree topology, Stone duality
06D22 Frames, locales
54D10 Lower separation axioms (T_0 – T_3 , etc.)

Cited in 1 Document

Keywords:

frame; $\mathbf{l}$ locale; sublocale; preframe; preframe homomorphism; weak homomorphism; binary coproduct of frames; diagonal map; strongly Hausdorff frame; fit frame; T_U -frame; simple extension

Full Text: DOI

References:

- [1] Banaschewski, B., Another look at the localic Tychonoff Theorem, Comment. Math. Univ. Carolin., 29, 647-656 (1988) · Zbl 0667.54009
- [2] Banaschewski, B., Singly generated frame extensions, J. Pure Appl. Algebra, 83, 1-21 (1992) · Zbl 0774.18004 · doi:10.1016/0022-4049(92)90101-K
- [3] Banaschewski, B.; Pultr, A., On weak lattice and frame homomorphisms, Algebra Univ., 51, 137-151 (2004) · Zbl 1079.06010 · doi:10.1007/s00012-004-1854-4
- [4] Clementino, M.M.: Separation and Compactness in Categories, Doctoral dissertation, University of Coimbra (1991)
- [5] Clementino, M.M.; Giuli, E.; Tholen, W., Topology in a category: compactness, Portugal. Math., 53, 397-433 (1996) · Zbl 0877.18002
- [6] Clementino, M.M., Giuli, E., Tholen, W.: A functional approach to general topology, In: Categorical Foundations, Encyclopedia Math. Appl., vol. 97, Cambridge University Press, Cambridge, (2004), pp. 103-163 · Zbl 1059.54012
- [7] Clementino, M.M., Picado, J., Pultr, A.: The other closure and complete sublocales, Appl. Categ. Structures 26, 892-906 (2018) corr. 907-908 · Zbl 06954308
- [8] Dowker, C.H., Strauss, D.: (T_1) - and (T_2) -axioms for frames, In: Aspects of Topology, London Math. Soc. Lecture Note Series, Vol. 93, Cambridge University Press, Cambridge (1985), pp. 325-335

- [9] Gutiérrez García, J., Kubiak, T., Picado, J.: On hereditary properties of extremally disconnected frames and normal frames, *Topology Appl.* 273 1-15 (2020) article no. 106978 · [Zbl 1477.06033](#)
- [10] Isbell, JR, Atomless parts of spaces, *Math. Scand.*, 31, 5-32 (1972) · [Zbl 0246.54028](#) · [doi:10.7146/math.scand.a-11409](#)
- [11] Isbell, JR, Function spaces and adjoints, *Math. Scand.*, 36, 312-339 (1975) · [Zbl 0309.54016](#) · [doi:10.7146/math.scand.a-11581](#)
- [12] Johnstone, PT, *Stone Spaces* (1982), Cambridge: Cambridge University Press, Cambridge · [Zbl 0499.54001](#)
- [13] Johnstone, P.T., Vickers, S.: Preframe presentations present, In: *Category Theory (Como, 1990)*, *Lecture Notes in Mathematics*, Vol. 1488, Springer, Berlin, pp. 193-212, (1991) · [Zbl 0764.18004](#)
- [14] Joyal, A., Tierney, M.: An extension of the Galois theory of Grothendieck, *Mem. Am. Math. Soc.* 51 (1984), no. 309 · [Zbl 0541.18002](#)
- [15] Manes, EG, Compact Hausdorff objects, *General Topol. Appl.*, 4, 341-360 (1974) · [Zbl 0289.54003](#) · [doi:10.1016/0016-660X\(74\)90011-7](#)
- [16] Picado, J.; Pultr, A., *Frames and Locales: Topology Without Points*, *Frontiers in Mathematics* (2012), Basel: Springer, Basel · [Zbl 1231.06018](#) · [doi:10.1007/978-3-0348-0154-6](#)
- [17] Picado, J.; Pultr, A., More on subfitness and fitness, *Appl. Categ. Struct.*, 23, 323-335 (2015) · [Zbl 1368.06005](#) · [doi:10.1007/s10485-014-9366-7](#)
- [18] Picado, J.; Pultr, A., *Separation in Point-free Topology* (2021), Cham: Birkhäuser-Springer, Cham · [Zbl 07261770](#) · [doi:10.1007/978-3-030-53479-0](#)
- [19] Picado, J.; Pultr, A., On equalizers in the category of locales, *Appl. Categ. Struct.*, 29, 267-283 (2021) · [Zbl 1467.18023](#) · [doi:10.1007/s10485-020-09616-8](#)
- [20] Townsend, C.: *Preframe techniques in constructive locale theory*. Department of Computing, Imperial College London (1996). (PhD Thesis)

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