

**Picado, Jorge; Pultr, Aleš**

**On strong inclusions and asymmetric proximities in frames.** (English) Zbl 1282.06013

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Authors' abstract: The strong inclusion, a specific type of subrelation of the order of a lattice with pseudocomplements, has been used in the concrete case of the lattice of open sets in topology for an expedient definition of proximity, and allowed for a natural pointfree extension of this concept. A modification of a strong inclusion for biframes then provided a pointfree model also for the non-symmetric variant. In this paper we show that a strong inclusion can be non-symmetrically modified to work directly on frames, without prior assumption of a biframe structure. The category of quasiproximal frames thus obtained is shown to be concretely isomorphic with the biframe based one, and shown to be related to that of quasi-uniform frames in a full analogy with the symmetric case.

Reviewer: **Tomasz Kubiak (Poznań)**

**MSC:**

**06D22** Frames, locales  
**06D15** Pseudocomplemented lattices  
**54E05** Proximity structures and generalizations

Cited in **3** Documents

**Keywords:**

frame; biframe; strong inclusion; quasi-uniform frame; quasi-proximal frame; total boundedness; pseudo-complement

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