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Monotone normality and stratifiability from a pointfree point of view. (English)

Zbl 1311.54014

Topology Appl. 168, 46-65 (2014).

Monotone normality has traditionally been studied in the class of T_1 -spaces. The restriction to this class is perhaps mainly due to the fact that every subspace of a monotonically normal T_1 -space is monotonically normal. In the article [*J. Gutiérrez García et al.*, Acta Math. Hung. 122, No. 1–2, 71–80 (2009; Zbl 1240.54077)], the authors study monotone normality in topological spaces which are not necessarily T_1 -spaces. They show, among other things, that in this broader context the property is no longer hereditary, in general. It is thus desirable to know if there is a separation axiom weaker than the T_1 -axiom under which monotone normality is a hereditary property. The authors of the paper under review answer this question most elegantly by stepping outside Top into the larger category Loc. It turns out that the separation axiom which does the trick is subfitness. The depth of the paper lies in the fact that the authors are able to simultaneously improve the classical results in the aforementioned paper, and extend them to the pointfree setting. Not only that; the paper also introduces and studies the natural localic counterpart of the notion of stratifiable spaces.

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MSC:

- 54D15 Higher separation axioms
- 06D22 Frames, locales
- 54C20 Extension of maps on topological spaces
- 54C99 Maps and spaces defined by maps

Keywords:

monotone normality; Borges operator; hereditary monotone normality; monotonically normal operator; stratifiability; subfit space; frame; locale; subfit frame; weakly subfit frame; open sublocale; closed map

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