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(Sub)fit biframes and non-symmetric nearness. (English) Zbl 1316.06012
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Nearnesses in pointfree topology are admitted precisely by regular frames. The initial approach to the concept of nearness in frames was via systems of covers, called uniform covers. This naturally made everything symmetric. The authors of the present paper discuss a non-symmetric variant via Weil entourages that were introduced to pointfree topology by the first-named author. Included in their tool kit are techniques from biframes. Starting by describing a sublocale of a biframe, and observing that each sublocale of a biframe is a biframe, they show that (i) a sublocale of a fit biframe is subfit, and (ii) a biframe is fit if and only if each of its sublocales is subfit. Whereas, as already mentioned, only regular frames can admit a nearness, the authors prove that a biframe admits a quasi-nearness precisely when it is subfit. Analogously with this result, they establish that a frame admits a quasi-nearness if and only if it is the total part of a subfit biframe.

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MSC:

[06D22](#) Frames, locales
[54D10](#) Lower separation axioms (T_0 – T_3 , etc.)
[54E55](#) Bitopologies
[54E15](#) Uniform structures and generalizations
[54E17](#) Nearness spaces

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Keywords:

[frames](#); [biframes](#); [locales](#); [sublocale lattices](#); [fitness](#); [subfitness](#); [pair-covers](#); [entourages](#); [nearness](#); [quasi-nearness](#)

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References:

- [1] Banaschewski, B.: Completion in pointfree topology, Lecture notes in mathematics and applied mathematics 2 (1996)
- [2] Banaschewski, B.; Brümmer, G. C. L.; Hardie, K. A.: Biframes and bispaces, Quaest. math. 6, 13-25 (1983) · [Zbl 0513.06005](#) · [doi:10.1080/16073606.1983.9632289](#)
- [3] Banaschewski, B.; Pultr, A.: Cauchy points of uniform and nearness frames, Quaest. math. 19, 101-127 (1996) · [Zbl 0861.54023](#) · [doi:10.1080/16073606.1996.9631828](#)
- [4] Dube, T.: Strong nearness frames, Proceedings symposium on categorical topology (Univ. Cape town, 1994), 103-112 (1999) · [Zbl 0986.54036](#)
- [5] Frith, J.: Structured frames, (1987)
- [6] Frith, J.; Schauerte, A.: The Samuel compactification for quasi-uniform biframes, Topol. appl. 156, 2116-2122 (2009) · [Zbl 1172.06005](#) · [doi:10.1016/j.topol.2009.03.034](#)
- [7] Frith, J.; Schauerte, A.: Quasi-nearnesses on biframes and their completions, Quaest. math. 33, 507-530 (2010) · [Zbl 1274.06041](#)
- [8] Frith, J.; Schauerte, A.: Quasi-nearness biframes: unique completions and related covering properties, Quaest. math. 35, 35-56 (2012) · [Zbl 1274.06042](#)
- [9] Gantner, T. E.; Steinlage, R. C.: Characterizations of quasi-uniformities, J. lond. Math. soc. 5, 48-52 (1972) · [Zbl 0241.54023](#) · [doi:10.1112/jlms/s2-5.1.48](#)
- [10] Herrlich, H.: A concept of nearness, Gen. topol. Appl. 5, 191-212 (1974) · [Zbl 0288.54004](#)
- [11] Herrlich, H.: Topologie II: Uniforme Räume, (1988) · [Zbl 0644.54001](#)
- [12] Herrlich, H.; Pultr, A.: Nearness, subfitness and sequential regularity, Appl. categ. Struct. 8, 67-80 (2000) · [Zbl 0978.54001](#) · [doi:10.1023/A:1008786312677](#)
- [13] Isbell, J. R.: Atomless parts of spaces, Math. scand. 31, 5-32 (1972) · [Zbl 0246.54028](#)

- [14] Johnstone, P. T.: Stone spaces, Cambridge studies in advanced mathematics 3 (1982)
- [15] Picado, J.: Weil uniformities for frames, *Comment. math. Univ. carol.* 36, 357-370 (1995) · [Zbl 0832.54025](#)
- [16] Picado, J.: Weil nearness spaces, *Port. math.* 55, 233-254 (1998) · [Zbl 0905.54017](#)
- [17] Picado, J.: Frame quasi-uniformities by entourages, *Proceedings symposium on categorical topology* (Univ. Cape town, 1994), 161-175 (1999) · [Zbl 1005.54030](#)
- [18] Picado, J.: Structured frames by Weil entourages, *Appl. categ. Struct.* 8, 351-366 (2000) · [Zbl 0965.06012](#) · [doi:10.1023/A:1008713430424](#)
- [19] Picado, J.; Pultr, A.: Locales treated mostly in a covariant way, *Textos de matemática* 41 (2008) · [Zbl 1154.06007](#)
- [20] Picado, J.; Pultr, A.: Cover quasi-uniformities in frames, *Topol. appl.* 158, 869-881 (2011) · [Zbl 1215.06005](#) · [doi:10.1016/j.topol.2011.01.007](#)
- [21] Picado, J.; Pultr, A.: On strong inclusions and asymmetric proximities in frames, *Order* 29, 513-531 (2012) · [Zbl 1282.06013](#)
- [22] Picado, J.; Pultr, A.: Frames and locales: topology without points, *Frontiers in mathematics* 28 (2012) · [Zbl 1231.06018](#)
- [23] Picado, J.; Pultr, A.: Entourages, covers and localic groups, *Appl. categ. Struct.* 21, 49-66 (2013) · [Zbl 1277.06005](#)
- [24] Pultr, A.: Pointless uniformities I, *Comment. math. Univ. carol.* 25, 91-104 (1984) · [Zbl 0543.54023](#)
- [25] Salbany, S.: A bitopological view of topology and order, *Sigma series in pure mathematics* 5, 481-504 (1984) · [Zbl 0557.54020](#)
- [26] Schauerte, A.: Biframes, (1992) · [Zbl 0817.06010](#)
- [27] Simmons, H.: The lattice theoretic part of topological separation properties, *Proc. edinb. Math. soc.* (2) 21, 41-48 (1978) · [Zbl 0396.54014](#) · [doi:10.1017/S0013091500015868](#)
- [28] Simmons, H.: Regularity, fitness, and the block structure of frames, *Appl. categ. Struct.* 14, 1-34 (2006) · [Zbl 1102.06008](#) · [doi:10.1007/s10485-005-9010-7](#)
- [29] Tukey, J. W.: Convergence and uniformity in topology, *Annals of mathematics studies* 2 (1940) · [Zbl 66.0961.01](#)
- [30] Weil, A.: Sur LES espaces à structure uniforme et sur la topologie générale, (1938) · [Zbl 0019.18604](#)