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Notes on exact meets and joins. (English) Zbl 1323.06007

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Exact meets in a lattice are special types of infima. Let L be a lattice. For any $a \in L$, set $\mathfrak{c}(a) = \{x \in L \mid x \geq a\}$. If L is a locale, then of course $\mathfrak{c}(a)$ is the closed sublocale determined by the element a. The authors call a subset S of L geometric if whenever $\bigwedge M$ exists, for $M \subseteq S$, then $\bigwedge M \in S$. They prove that the lattice $\mathcal{G}(L)$ of geometric subsets of L is complete, and then obtain, among others, an internal characterisation of exact meets as precisely those $\bigwedge A$ for which $\bigvee_A \mathfrak{c}(a) = \mathfrak{c}(b)$, for some $b \in L$, where the join is calculated in $\mathcal{G}(L)$.

Specialising to frames, more results come to the fore. For instance, a meet $a = \bigwedge_i a_i$ is exact if and only if the double co-pseudocomplement (calculated in the coframe of sublocales) of the sublocale $\bigcap_i \mathfrak{o}(a_i)$ is the open sublocale $\mathfrak{o}(a)$. Weakening exactness somewhat, the authors introduce a notion they call cozero exactness. They then use that to characterise P-frames as precisely those in which countable meets are cozero exact. This characterisation seeks to mimic the characterisation of P-spaces as those in which every G_δ-set is open.

A stronger property is what *J. Todd Wilson* [The assembly tower and some categorical and algebraic aspects of frame theory, PhD thesis, Carnegie Mellon University (1994; unpublished)] calls free meets. These are meets that are preserved by all frame homomorphisms. In the current paper the authors call them strong exact meets. They characterise them in a number of ways. Another specialisation the authors undertake is a study of exact meets in spaces and spatial frames.

The authors state in the abstract that the paper studies these concepts systematically. Indeed, not only is the study systematic, it is very thorough to boot.

Reviewer: Themba Dube (Unisa)

MSC:

06D22	Frames, locales	Cited in **1** Document
18B35	Preorders, orders and lattices (viewed as categories)	
54D10	Lower separation axioms (T_0–T_3, etc.)	

Keywords:

frames; locales; sublocales; lattices; exact meets; exact joins; free meets; T_D-topological spaces; Scott topology; P-frames; paracompact frames

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