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Continuous extensions of real functions on arbitrary sublocales and C -, C^* -, and z -embeddings. (English) Zbl 1473.18014

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The sublocale lattices are much more complicated than their topological counterparts (complete atomic Boolean algebras). Some of the main differences are that (i) each locale has a smallest dense sublocale, (ii) only complemented sublocales (most sublocales are not complemented) distribute over all covers, and (iii) covers are not necessarily stable under pullbacks.

S. Mrowka [Nieuw Arch. Wiskd., III. Ser. 16, 94–111 (1968; [Zbl 0183.40901](#))] presented his interesting **Topological Extension Theorem** to extend a continuous function from an arbitrary subspace to the whole space. A point-free counterpart of the extension theorem of Mrówka was proved by *J. Gutiérrez García* and *J. Picado* [Topology Appl. 158, No. 17, 2264–2278 (2011; [Zbl 1238.06008](#))] for a complemented sublocale of a locale.

The main purpose of the authors of this paper is to present a proof of that theorem for arbitrary sublocales.

Finally, after recalling that a sublocale S of L is said to be C -embedded (resp. C^+ -embedded) if every f in $\mathcal{C}(S)$ (respectively in $\mathcal{C}^+(S)$) has a continuous extension (resp. bounded continuous extension) to L , the authors revisit the theory C - and C^+ -quotients of [*R. N. Ball* and *J. Walters-Wayland*, Diss. Math. 412, 62 p. (2002; [Zbl 1012.54025](#))] from the point of view of sublocale embeddings. In addition, they also treat the parallel class of z -embedded sublocales studied in [*J. Gutiérrez García* et al., “Notes on point-free real functions and sublocales”, Textos de Matem. DMUC 46, 167–200 (2014)] in order to get some applications of their new results.

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MSC:

[18F70](#) Frames and locales, pointfree topology, Stone duality
[06D22](#) Frames, locales
[54C20](#) Extension of maps
[54C30](#) Real-valued functions in general topology
[54C45](#) C - and C^* -embedding

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Keywords:

[frame](#); [locale](#); [completely separated sublocales](#); [C- and C*-embedded sublocales](#); [z-embedded sublocale](#); [localic extension theorem](#)

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