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Notes on point-free topology. (English) [Zbl 1486.18018](#)

Clementino, Maria Manuel (ed.) et al., New perspectives in algebra, topology and categories. Summer school, Louvain-la-Neuve, Belgium, September 12–15, 2018 and September 11–14, 2019. Cham: Springer. Coimbra Math. Texts 1, 173–223 (2021).

These notes cover the topics presented by the first author in his course on *Frames and Locales* at the *Summer School in Algebra and Topology*.

After remembering that topology can be cultivated as the natural geometry of places (“spots”) and their interrelations, that we do not have to think of them as sets of points and that they can be thought as entities in their own right, the authors achieve an excellent presentation of the basic ideas and motivation for point-free thinking, to illustrate the reasoning (for this, they present some proofs hoping to persuade the reader that the techniques are in fact fairly friendly) and present some results to illustrate the merits of the approach.

The first and most important fact is their exposition of point-free topology as a natural synthetic general geometry and briefly outline the history; secondly, they discuss the necessary order-theoretic background, the relation to classical spaces, and some categorical aspects; thirdly, they introduce the reader to some concrete facts about point-free spaces (frames and locales), finally, the authors present a few facts showing the advantages and merits of this way of viewing point-free topology.

For the entire collection see [\[Zbl 1475.18001\]](#).

Reviewer: [Joaquín Luna-Torres \(Cartagena\)](#)

MSC:

- [18F70](#) Frames and locales, pointfree topology, Stone duality
- [06D22](#) Frames, locales
- [18-02](#) Research exposition (monographs, survey articles) pertaining to category theory
- [54-02](#) Research exposition (monographs, survey articles) pertaining to general topology

Keywords:

[point-free topology](#); [category of frames](#); [category of locales](#); [Heyting algebra](#); [sober space](#)

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