

Isometry Groups as Topological Groups

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Metric geometry is an active field of mathematical research with numerous impressive results and tons of open questions. This fascinating area of mathematics is connected to the world of topological algebra via the isometry group:

Given a metric space (X, d) , the group of isometric bijections $\text{Iso}(X, d)$ can be endowed with a natural topology, turning it into a topological group. It can be shown that every complete topological group can be realized as the isometry group of a suitable metric space, so the isometry group $\text{Iso}(X, d)$ of a general metric space (X, d) can be very complicated in general. Therefore, it is a natural question under which additional assumptions on the space (X, d) we obtain more well-behaved topological groups as $\text{Iso}(X, d)$.

There are many results in that direction, for example $\text{Iso}(X, d)$ is a Lie group if (X, d) is a Riemannian manifold, endowed with its standard distance function.

In this talk, I will give a survey about isometry groups and how geometric properties of the space correspond to topological and algebraic properties of the group. I will also address an interesting open question about certain infinite-dimensional metric spaces and pro-Lie groups (in the sense of Hofmann/Morris).

*This is joint work with Artem Nepechiy (KIT).