Interrogative agendas and decision making

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Krishna Manoorkar
joint ongoing work with Marcel Boersma, Alessandra Palmigiano, Apostolos Tzimoulis, , Nachoem Wijnberg, and more

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A deliberation scenario (Baltag et al. 2017)

John and Mary: candidates for an open position in Philosophy/Logic.
John has better letters of reference than Mary.
John's philosophical writing is slightly better than Mary's, but his formal proofs are full of mistakes.
Mary's logical work is high-quality and fully backs her philosophical claims.

	References	Philosophy	Logic
John	1	1	0
Mary	0	1	1

Alan and Betty: members of hiring committee. Alan (a) is philosophy expert, does not understand formal logic.
Betty (b) is a formal logician, not really concerned with philosophy.
Winner: candidate who performs equally well or better on (all and only) the issues which a and b agree to be relevant.

2

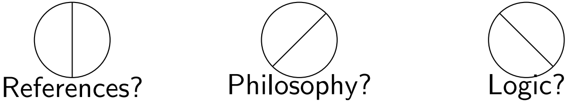
Main aim

- Formal framework to describe and reason about the essentials of **deliberation processes**;
- **Dynamic** representation;
- Similar but **different** from preference/judgment aggregation:
- **Predicting outcomes** of deliberation processes based on:
  - agendas of agents;
  - axioms of interactions:

Interrogative agendas

The ‘conjunction’ of the **issues** considered relevant by an agent/group of agents.

- Issues as yes/no questions:



- Which issues are relevant for whom?
  Alan and Betty’s **interrogative agendas**:

Interrogative agendas

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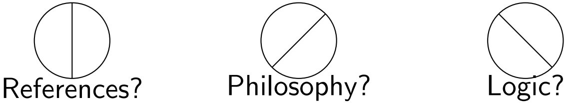
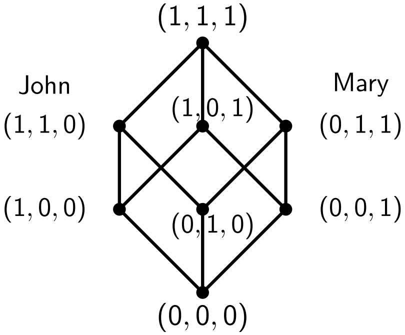
- Issues as yes/no questions:



- Which issues are relevant for whom?
  Alan and Betty's **interrogative agendas**:

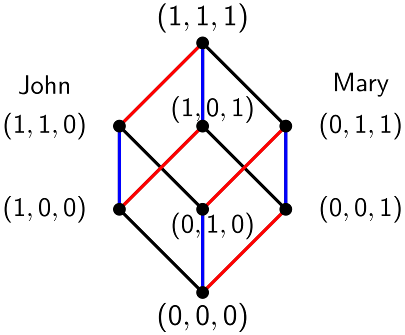
What is the space being partitioned here?

$\{0,1\}$-valued feature spaces



The **winning rule** induces a natural **preference (pre-)order**

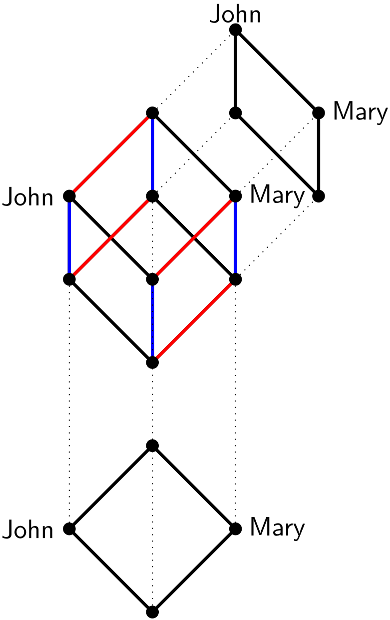
$\{0,1\}$-valued feature spaces



Alan and Betty's interrogative agendas

as equivalence relations/partitions

$\{0, 1\}$-valued feature spaces



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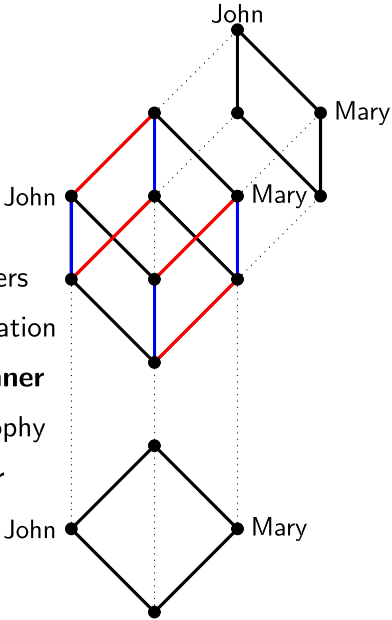
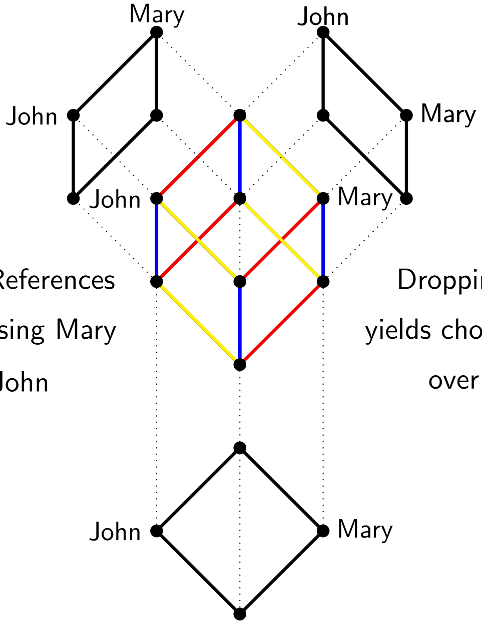


All three parameters
yield the identity relation
which gives **no winner**
as dropping Philosophy
gives **no winner**

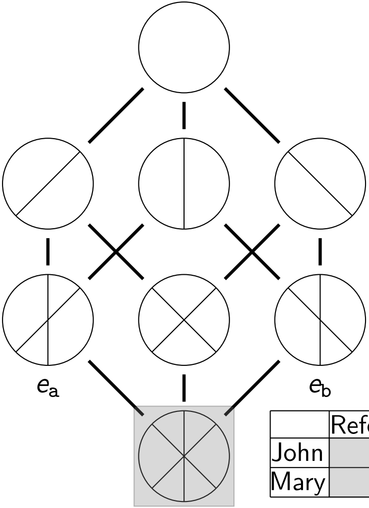
$\{0,1\}$-valued feature spaces



Dropping References
yields choosing Mary
over John

Dropping Logic
yields choosing John
over Mary

Meet-semilattice generated by relevant issues



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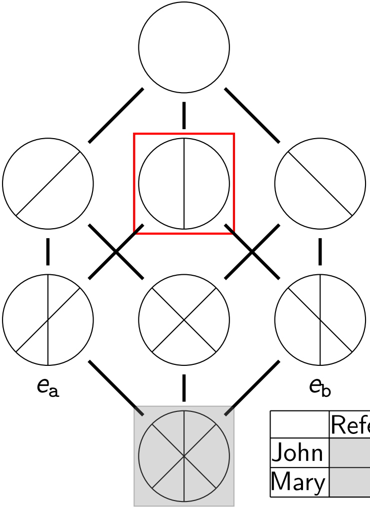

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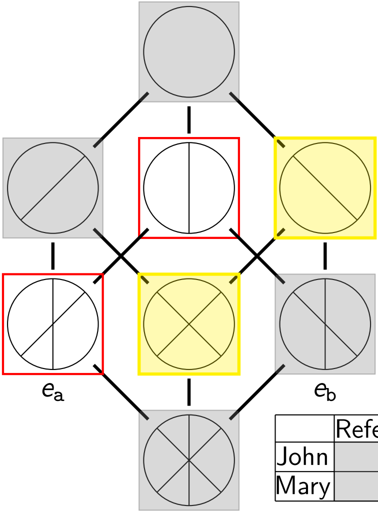

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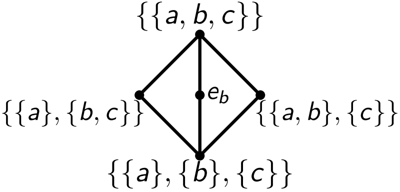
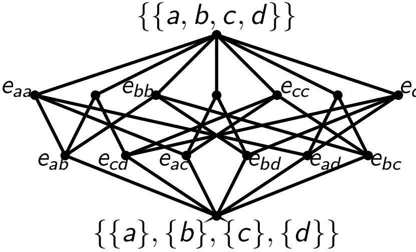
The lattice of equivalence relations over a set

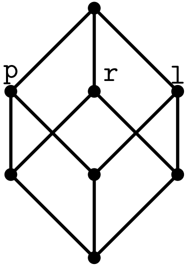
$E(W)$ for $W := \{a, b, c\}$ and $W := \{a, b, c, d\}$





- These lattices are in general non-distributive but like power-set algebras they are completely join-generated and meet-generated by their atoms and co-atoms.

Multi-type models for deliberation



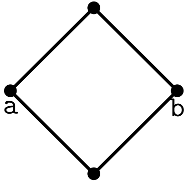
$\mathbb{D}$

algebra of

interrogative agendas

meet-generated

by issues



$\mathbb{C}$

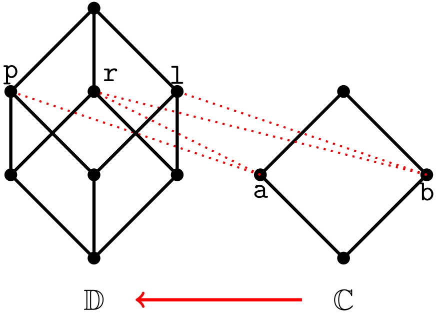
algebra of

'coalitions'

join-generated

by agents

Multi-type models for deliberation

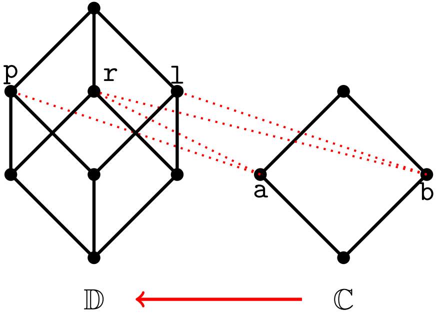


$R \subseteq M^{\infty}(\mathbb{D}) \times J^{\infty}(\mathbb{C})$ $\quad$ $\mathtt{m}R\mathtt{j}$ iff issue $\mathtt{m}$ **relevant** to agent $\mathtt{j}$

$\Diamond : \mathbb{C} \to \mathbb{D}$, $\quad \Diamond c :=$ **common agenda** of c

$\rhd : \mathbb{C} \to \mathbb{D}$, $\quad \rhd c :=$ **distributed agenda** of c

Multi-type models for deliberation



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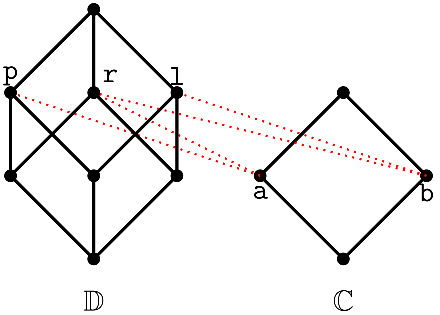
$\blacksquare : \mathbb{D} \to \mathbb{C}$, $\quad \blacksquare e :=$ **largest coalition $\forall$-supporting** e

$\blacktriangleright : \mathbb{D} \to \mathbb{C}$, $\quad \blacktriangleright e :=$ **largest coalition $\exists$-supporting** e

$\Diamond\blacksquare : \mathbb{D} \to \mathbb{D}$, $\quad \Diamond\blacksquare e :=$ issues going 'in a package'

$\blacksquare\Diamond : \mathbb{C} \to \mathbb{C}$, $\quad \blacksquare\Diamond c :=$ 'people who like this also like...'

Multi-type models for deliberation



$S \subseteq M^\infty(\mathbb{D}) \times J^\infty(\mathbb{C}) \times M^\infty(\mathbb{D}) \quad S(\mathtt{n},\mathtt{i},\mathtt{m})$ iff

agent $\mathtt{i}$ is willing to **substitute** issue $\mathtt{m}$ with issue $\mathtt{n}$

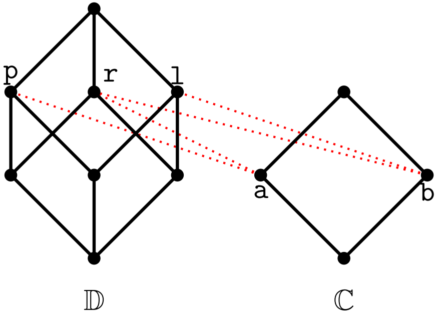
$$-\!\!< \,:\mathbb{C} \times \mathbb{D} \to \mathbb{D}$$

$c -\!\!< e :=$ common view in c on how to modify e

$$\star : \mathbb{D} \times \mathbb{D} \to \mathbb{C} \quad -\!\!<' : \mathbb{C} \times \mathbb{D} \to \mathbb{D}$$

$$\vartriangleright : \mathbb{C} \times \mathbb{D} \to \mathbb{D} \quad \blacktriangleright : \mathbb{D} \times \mathbb{D} \to \mathbb{C} \quad \vartriangle : \mathbb{C} \times \mathbb{D} \to \mathbb{D}$$

Examples



$$S_1 := \{(\mathtt{p},\mathtt{a},\mathtt{p}),(\mathtt{r},\mathtt{a},\mathtt{r}),(\mathtt{p},\mathtt{a},\mathtt{l}),(\mathtt{r},\mathtt{a},\mathtt{l}),(\mathtt{r},\mathtt{b},\mathtt{r}),(\mathtt{l},\mathtt{b},\mathtt{l}),(\mathtt{r},\mathtt{b},\mathtt{p}),(\mathtt{l},\mathtt{b},\mathtt{p})\}.$$

$$S_2 := \{(\mathtt{p},\mathtt{a},\mathtt{p}),(\mathtt{r},\mathtt{a},\mathtt{r}),(\mathtt{l},\mathtt{a},\mathtt{r}),(\mathtt{l},\mathtt{a},\mathtt{l}),(\mathtt{l},\mathtt{b},\mathtt{l}),(\mathtt{r},\mathtt{b},\mathtt{r}),(\mathtt{p},\mathtt{b},\mathtt{r}),(\mathtt{p},\mathtt{b},\mathtt{p})\}$$

Very preliminary end of deliberation story

Outcome of deliberation: $(\mathtt{a} \multimap\!< \Diamond \mathtt{b}) \sqcap (\mathtt{b} \multimap\!< \Diamond \mathtt{a})$

- $\multimap\!<_{S_1} : \mathbb{C} \times \mathbb{D} \to \mathbb{D}$ is such that

$$\mathtt{a} \multimap\!< \Diamond \mathtt{b} = \mathtt{a} \multimap\!< (\mathtt{r} \sqcap \mathtt{l}) = (\mathtt{a} \multimap\!< \mathtt{r}) \sqcup (\mathtt{a} \multimap\!< \mathtt{l}) = \mathtt{r} \sqcup (\mathtt{r} \sqcap \mathtt{p}) = \mathtt{r}$$

$$\mathtt{b} \multimap\!< \Diamond \mathtt{a} = \mathtt{b} \multimap\!< (\mathtt{r} \sqcap \mathtt{p}) = (\mathtt{b} \multimap\!< \mathtt{r}) \sqcup (\mathtt{b} \multimap\!< \mathtt{p}) = \mathtt{r} \sqcup (\mathtt{r} \sqcap \mathtt{l}) = \mathtt{r}$$

  Hence, outcome of deliberation is $\mathtt{r}$, yielding John over Mary.

- $\multimap\!<_{S_2} : \mathbb{C} \times \mathbb{D} \to \mathbb{D}$ is such that

$$\mathtt{a} \multimap\!< \Diamond \mathtt{b} = \mathtt{a} \multimap\!< (\mathtt{r} \sqcap \mathtt{l}) = (\mathtt{a} \multimap\!< \mathtt{r}) \sqcup (\mathtt{a} \multimap\!< \mathtt{l}) = (\mathtt{r} \sqcap \mathtt{l}) \sqcup \mathtt{l} = \mathtt{l}$$

$$\mathtt{b} \multimap\!< \Diamond \mathtt{a} = \mathtt{b} \multimap\!< (\mathtt{r} \sqcap \mathtt{p}) = (\mathtt{b} \multimap\!< \mathtt{r}) \sqcup (\mathtt{b} \multimap\!< \mathtt{p}) = (\mathtt{r} \sqcap \mathtt{p}) \sqcup \mathtt{p} = \mathtt{p}$$

  Hence, outcome of deliberation is $\mathtt{p} \sqcap \mathtt{l}$, yielding Mary over John.

Multi-type correspondence

- S symmetric iff $e_1 \star e_2 \leq e_2 \star e_1$ valid iff $c \,{-\!\!\prec}'\, e \leq c \,{-\!\!\prec}\, e$ valid.
- S is *positively coherent with* R if

$$\forall \mathtt{j} \forall \mathtt{m} [\mathtt{m} R \mathtt{j} \Rightarrow S(\mathtt{m}, \mathtt{j}, \mathtt{m})].$$

- S positively coherent with R iff $\rhd c \succ\!\!- e \leq c \triangle e$ valid, iff $c \unrhd e \leq \rhd c \sqcup e$ valid.
- Transitivity of S is not modally definable.

Conclusions and Future Works

- it's all modal logic:
- insights and results from modal logic transfer smoothly to multi-type;
- relational semantics, algebra and proof calculi from general theory;
- Unified correspondence, algebraic proof theory, Goldblatt-Thomason.

Formal Concept Analysis

A **formal context** is a tuple $\mathbb{P} = (A, X, I)$, where A, X are interpreted as sets of **objects** and **features** and relation I is interpreted as
aIx iff object a has feature x.

- $I^{(1)}[B] = \{x \in X \mid a \in B \implies aIx\}$ is the set of features shared by all objects of B
- $I^{(0)}[Y] = \{a \in A \mid x \in Y \implies aIx\}$ is the set of objects having all the features in Y.
- These operators form a **Galois connection**.

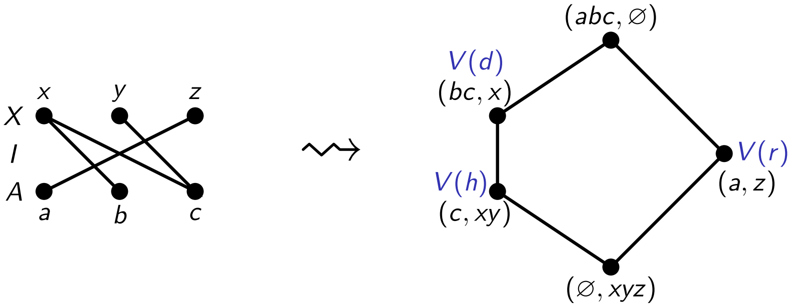
The Galois-stable sets of objects form (well defined) **concepts** or meaningful categorizations of X. A concept of $\mathbb{P}$ is any pair $(B, I^{(1)}[B])$, where B is Galois-stable.

- By **Birkhoff's theorem** concepts of $\mathbb{P}$ form a complete lattice ($\mathbb{P}^+$) called **concept lattice** of $\mathbb{P}$.

Concept Lattice

$a =$ *A Midsummer Night's Dream*
$b =$ *King Lear*
$c =$ *Julius Caesar*

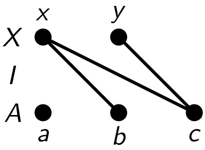
$x =$ *'no happy end'*,
$y =$ *'real historical figures'*,
$z =$ *'two characters fall in love'*



$r =$ 'romantic comedy', $d =$ 'drama' $h =$ 'historical drama'.

Agendas and categorization

In case y is not relevant feature for us.



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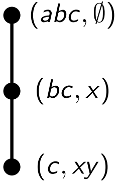

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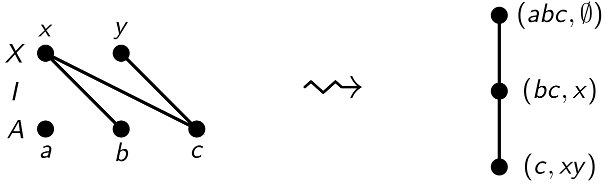


Desired or required categorization depends on agendas of interest.

Formal concept analysis and agendas

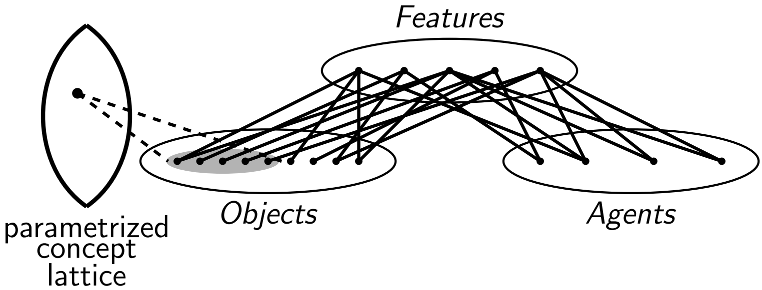
An agenda for categorization is given by $Y \subseteq X$. Intuitively this corresponds to $Y \subseteq X$ being the "**features of interest**" for a specific categorization.

Definition

Formal context (or categorization) induced by agenda Y to be $(A, Y, I \cap A \times Y)$ and **induced categorization** is given by corresponding concept lattice.

- Induced categorizations form a lattice under the order given by inclusion of feature sets.
- Thus, agendas of different agents induce different categorizations.

Categorizations based on agendas and interaction



Future directions

- Extending to non-crisp cases - Dempster-Shafer theory
- Learning agendas