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Generalized Eilenberg Theorem

Eilenberg proved that varieties of finite monoids bijectively correspond to varieties of regular languages, i.e., classes of regular languages closed under the boolean set-theoretical operations, derivations, and preimages under monoid homomorphisms. We prove a much more general result, based on combining coalgebraic and algebraic methods.

We work with a locally finite variety C of algebras (instead of just boolean algebras). Then we form the predual category D which means that D is the ind-completion of the dual of all finitely presentable objects of C. The role of finite monoids is now taken by finite bimonoids in D. Example: if C are distributive lattices, then D are posets. We thus prove the result of [2] that varieties of finite ordered monoids bijectively correspond to lattice-varieties of regular languages (closed under union and intersection but not necessarily under complement). Another example: if C are vector spaces over the binary field, then D equals C, and the role of finite monoids is taken over by algebras over the field (in the classical sense of K-algebras). We thus prove that varieties of finite K-algebras bijectively correspond to vector-varieties of regular languages (closed, instead of under boolean operations, under symmetric difference).

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*Joint work with S. Milius, R. Myers and H. Urbat.

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Distributivity of convex lattices

It is well-known that all vector lattices are distributive lattices; so this purely lattice theoretical property is a consequence of linearity. So the question is whether some weaker lattices with some more general convexity structure are also distributive (or maybe modular). Such abstract convex or absolutely convex spaces form an Eilenberg-Moore category over the category of sets; they were studied by D. Pumplün and H. Röhrl [3]. As F.E.J. Linton pointed out, they need not be (absolutely) convex subsets of vector spaces. Lattice structures on such absolutely convex spaces were first studied by S. Nörtemann in his PhD thesis [2], who raised the question whether such lattices are always distributive.

In her Diplomarbeit [4] J. Winzenick showed that lattices with an abstract convex structure need not be distributive. Moreover, every such convex space can be embedded into an absolutely convex one, i.e. adding an origin and negatives does not yield new equalities between the old elements. So absolutely convex lattices need not be distributive either. But all the constructions are done in abstract convex spaces, in which all points in the interior of a chord collapse. Such convex spaces were called *discrete* by N. Meier and studied in his PhD thesis [1]; they generalize Linton's idea. Moreover, every discrete modular lattice is distributive. Discreteness plays an important role in the proofs and simplifies things a lot; without this assumption most questions are still open.

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*Joint work with Jennyfer Winzenick.

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Mal'tsev reflection

It was showed in [1] that the fibers of the fibration $(\ )_0 : Grd\mathbb{E} \to \mathbb{E}$ associated with the internal groupoids in $\mathbb{E}$ are protomodular. No similar structural result existed for the fibers of the fibration $(\ )_0 : Cat\mathbb{E} \to \mathbb{E}$ associated with the internal categories in $\mathbb{E}$. A recent work about the category Mon of monoids [2], which is nothing but the fibre of this fibration above the singleton 1, focused our attention on classes of split epimorphisms between monoids (called Schreier, left homogeneous and homogeneous split epimophisms) satisfying partial aspects of the Mal'tsev and protomodular processes and properties. So it is quite natural to investigate whether the fibers $Cat_Y\mathbb{E}$ would not satisfy some property of this kind. This would be all the more interesting since it would produce a similar conceptual situation in a non-pointed context and would clarify the underlying relationship with Mal'tsevness and protomodularity which are not pointed concepts by themselves.

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Extension theory and the calculus of butterflies

Let $\mathcal{C}$ be a semi-abelian category satisfying the condition (SH) (i.e. where two equivalence relation centralize each other as soon as their normalizations commute). We give a cohomological classification of the extensions of an internal crossed module in $\mathcal{C}$ via a given object. More precisely, given an internal crossed module $(\partial\colon K \to K_0, \xi)$ and a morphism $\phi\colon Y \to \pi_0(\partial) = \mathrm{Coker}(\partial)$, we show that the set $\mathrm{Ext}_\phi(Y, \partial)$ of extensions (i.e. short exact sequences) (f, k) filling the following diagram (with $(1_K, \alpha)$ a crossed module morphism)

$$\begin{array}{ccc} K & = & K \\ {\scriptstyle k}\,\downarrow & & \downarrow\,{\scriptstyle \partial} \\ X & \overset{}{\underset{\alpha}{\dashrightarrow}} & K_0 \\ {\scriptstyle f}\,\downarrow & & \downarrow\,{\scriptstyle \mathrm{coker}(\partial)} \\ Y & \underset{\phi}{\longrightarrow} & \pi_0(\partial) \end{array}$$

either is empty, or it is a simply transitive $\mathrm{H}^2_{\overline{\phi}}(Y, \pi_1(\partial))$-set, where $\pi_1(\partial) = \mathrm{Ker}(\partial)$ is a Y-module with the action $\overline{\phi}$ induced by ξ.

The main tool we use is the calculus of *butterflies*, introduced by B. Noohi [5] to deal with monoidal functors between 2-groups and further developed in the semi-abelian context in [1], where the authors show that they are the bicategory of fractions of internal crossed modules with respect to weak equivalences.

The present result is an intrinsic version of a theorem by P. Dedecker [4] (stated in the category of groups) and extends, in the semi-abelian setting, the intrinsic version (developed in [2] and [3]) of the classical Schreier-Mac Lane Theorem on the classification of extensions.

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*Joint work with Giuseppe Metere.

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Enriched Morita equivalence for S-sorted theories

The theory of Morita equivalence characterises those pairs of algebraic theories that yield equivalent categories of algebras. The notion of a theory and of its algebras can be varied and many similar results arise: taking *rings* as theories and *modules* over a given ring as algebras, we get the original result of Morita [1]. The non-additive version (by Knauer and Banaschewski [2,3]) takes *monoids* as theories and *monoid actions* as algebras. Morita equivalent *Lawvere theories* were characterised by Dukarm [4]. For *many-sorted* algebraic theories, Adámek, Sobral and Sousa [5] proved a generalisation of Dukarm's result.

All these results are stated using the notion of a *pseudoinvertible idempotent*: two theories $\mathcal{T}'$ and $\mathcal{T}$ over the same set of sorts are Morita equivalent iff $\mathcal{T}'$ is an idempotent modification of $\mathcal{T}$, given some choice of pseudoinvertible idempotents in $\mathcal{T}$.

This gives a hint that it should be possible to state a more general result subsuming all the aforementioned results. We show that this is in fact true and we give a characterisation of S-sorted Morita equivalent theories (parametric in the choice of the notion of a theory) that works for ordinary categories as well as for enriched categories. We show some examples that abound naturally as a consequence of the main result.

This work is partially supported by the Grant Agency of the Czech Technical University in Prague, grant No. SGS13/069/OHK3/1T/13.

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*Joint work with Jiří Velebil.

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A classification theorem for normal extensions

In [4], a generalized Galois theorem has been proved in the large context of so-called admissible Galois structures. These are adjunctions $\langle I, H\rangle \colon \mathscr{C} \rightharpoonup \mathscr{X}$ with classes of morphisms ("extensions") $\mathcal{E}$ and $\mathcal{Z}$ (of $\mathscr{C}$ and $\mathscr{X}$, respectively) satisfying suitable properties. In my talk, I shall explain how we can obtain a similar classification theorem for normal extensions. For this, we essentially use descent theory (as presented in [6]) and work with a replacement of the admissibility condition which holds in many algebraic contexts: i.e. I preserves pullbacks

$$\begin{array}{ccc} A & \longrightarrow & B \\ \downarrow & & \downarrow g \\ D & \xrightarrow[h]{} & C \end{array}$$

with g, h in $\mathcal{E}$ and g a split epimorphism. Actually, this condition (already considered in [5, 2, 1, 3] for instance) provides the existence of a normalisation functor and good stability properties of the class of trivial extensions which are needed to get weakly universal normal extensions used in the theorem. Along the way, we show that the normalization functor is the pointwise Kan extension of a (restricted) trivialization functor.

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*Joint work with Tomas Everaert.

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A Galois-theoretic approach to the covering theory of quandles

The purpose of this work is to clarify the relationship between the algebraic notion of quandle covering introduced by M. Eisermann [1] and the categorical notion of covering arising from Galois theory [3]. A crucial role is played by the adjunction between the variety of quandles and its subvariety of trivial quandles.

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Graphs, polarities and completions of lattices

In [3] and [2] a one-to-one correspondence between the category of perfect lattices and the category of particular polarities, the so-called RS frames, is established. Since the canonical extensions of bounded lattices are perfect lattices, we may associate to any bounded lattice **L** the RS frame of its canonical extension, which sets are formed respectively by the completely join-irreducible and the completely meet-irreducible elements of the canonical extension. Such an RS frame can be obtained from a graph determined by **L**. This graph has particular properties which we use to define the category of TiRS graphs. The properties of these graphs can be translated into properties of polarities, giving rise to TiRS frames, which are RS frames satisfying an additional property. We prove that the category of TiRS graphs and the category of TiRS frames are equivalent. The RS frames associated to canonical extensions are TiRS frames and consequently can be obtained from TiRS graphs.

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*Joint work with A.P.K. Craig and M. Haviar.

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Semi-localizations of semi-abelian categories

A semi-localization of a category is a full reflective subcategory with the property that the reflector is a semi-left-exact functor. In this article we first determine an abstract characterization of the categories which are semi-localizations of an exact Mal'tsev category, by particularizing a result due to S. Mantovani [1]. We then turn our attention to semi-abelian categories, where a special type of semi-localizations are known to correspond to torsion theories [2]. For this purpose a new characterisation of protomodular categories is obtained, on the model of the one discovered by Z. Janelidze in the pointed context [3]. Both the torsion-free and the hereditarily-torsion-free subcategories of semi-abelian categories are then characterized, and some examples are examined in detail. We finally explain how these results extend similar ones obtained by W. Rump in the abelian context [4].

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*Joint work with Stephen Lack.

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On monads of (the dual) Kock-Zöberlein type in topology

A monad (T, m, e) on an order-enriched category is of Kock-Zöberlein type if T is locally monotone and $Te_X \leq e_{TX}$. This property of a monad is very convenient since it allows a description of their algebras as precisely the injectives with respect to a certain class of morphisms. There is an extensive literature on this type of monads on Top (typically submonads of the filter monad); however, monads satisfying the dual condition $Te_X \geq e_{TX}$ seem to be less frequent in topology. Having as starting point the Vietoris monad, in this talk we will have a closer look at some of these. If time permits, we will also give a characterisation of the morphisms of the Kleisli category of the Vietoris monad, and show how the notion of an Esakia space arises naturally in this context via splitting idempotents.

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2-Dimensional non-pointed exactness structures and radicals in categories

This work is devoted to non-pointed versions of constructions and results of [3], which at the same time is a 2-dimensional continuation of the radical theory proposed in [2] and reported on the First Workshop on Categorical Methods in Non-Abelian Algebra in Coimbra last year. It is also related to the approach to radical theory developed in [4]. "2-Dimensional" refers to the simplicial structure involved, and 2-dimensional simplexes represent non-pointed short exact sequences in the sense of M. Grandis (see [1] and references there). In particular, using this 2-dimensional structure allows to improve the connection between radicals and closure operators established in [2].

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*Joint work with M. Grandis and L. Márki.

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A survey of recent results on normal categories

A normal category, in the sense of [5], is a pointed regular category [1] in which every regular epimorphism is a normal epimorphism. A normal category can be also equivalently defined by replacing each occurrence of “regular epimorphism” in the definition of a regular category with “normal epimorphism” (and requiring the category to be pointed). Algebraic normal categories were first considered in [2]. Normal categories also implicitly arise in the “old era” of axiomatic investigations of categories of group-like structures, and more explicitly in the modern “new era” of these investigations (see e.g. [4], [6], [3], [7]). In this talk we recall some of the recent results on normal categories, present some new ones, and mention one or two open questions.

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A categorical model for 2-PDAs with states

Having recently established that deterministic push-down automata with two stacks (2PDAs) and a single state suffice to simulate Turing machines, we now turn to describing a categorical model for such machines, even those with states.

Recall that labeled transition systems (LTSs), which equipped with initial and final states and under suitable finiteness constraints form the basis for finite automata, can be viewed either as graph morphisms into a fixed target graph Σ (usually a graph with just a single node, in case of formal language theory), or, alternatively, as a graph morphism from Σ into the 2-category **Rel** of sets and relations.

In 1989, Bob Walters used morphisms of multigraphs into a specific target to categorify a certain variant of context-free grammars (sufficiently strong to capture all context-free languages).

By looking at a slightly larger class of context-free grammars and changing the point of view from multigraphs to co-multigraphs (cm-graphs, for short), a connection can be made between these context-free grammars (CFGs) and certain restricted push-down automata (PDAs). The latter may be viewed as CFGs with the additional constraint that rewriting can only happen at the end of the stack, the "current position", hence they are not very interesting for computer scientists.

However, when two stacks are involved, which we think of as being juxtaposed at their respective current positions, the possibility of *moving* this position is not only a very powerful extension to the capabilities of ordinary PDAs, but in addition has a very succinct categorical interpretation in terms of adjoints in cm-graphs. The previous observation that states and storage are orthogonal concepts for 2PDAs (not so for Turing machines!) then is reflected in the step from cm-graphs to fc-cm-graphs (a kind of double graphs analogous to fc-multicategories). Hence, from a rather different angle we arrive at a model with close connections to the *tile model* of Gadducci and Montanari.

It remains to be seen whether the alternative view of LTSs (as graph morphisms into **Rel**) can be evolved in a similar fashion.

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Push forwards of crossed squares

It is well known that given a crossed module $\partial : G_1 \to G_0$ of groups, then:

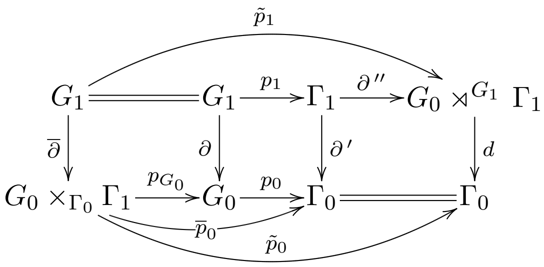
$\ker\partial$ is G_0-invariant, so that $\ker\partial \to G_0$ is a crossed module;

the action of G_0 on the abelian group $\ker\partial$ passes to $\operatorname{coker}\partial$ so that $\ker\partial \to \operatorname{coker}\partial$ is still a crossed module.

We show that there is a corresponding result if we start with a crossed square (an internal crossed module in the category of crossed modules):

$$\begin{array}{ccc} G_1 & \xrightarrow{p_1} & \Gamma_1 \\ {\scriptstyle\partial}\downarrow & & \downarrow{\scriptstyle\partial'} \\ G_0 & \xrightarrow{p_0} & \Gamma_0 \end{array}$$

and we take the homotopical version of kernels and cokernels, using pullbacks for the first and push forwards for the second, so that in the diagram



both $(p_1, \overline{p}_0)$ and $(\tilde{p}_1, \tilde{p}_0)$ give rise to crossed squares.

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*Joint work with L. Pizzamiglio.

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Commutative orders in semigroups

We consider commutative orders, that is, commutative semigroups having a semigroup of fractions in a local sense defined as follows. An element $a \in S$ is *square-cancellable* if for all $x, y \in S^1$ we have that $xa^2 = ya^2$ implies $xa = ya$ and also $a^2x = a^2y$ implies $ax = ay$. It is clear that being square-cancellable is a necessary condition for an element to lie in a subgroup of an oversemigroup. In a commutative semigroup S, the square-cancellable elements constitute a subsemigroup $\mathcal{S}(S)$. Let S be a subsemigroup of a semigroup Q. Then S is a *left order* in Q and Q is a *semigroup of left fractions* of S if every $q \in Q$ can be written as $q = a^\sharp b$ where $a \in \mathcal{S}(S)$, $b \in S$ and $a^\sharp$ is the inverse of a in a subgroup of Q and if, in addition, every square-cancellable element of S lies in a subgroup of Q. *Right orders* and *semigroups of right fractions* are defined dually. If S is both a left order and a right order in Q, then S is an *order* in Q and Q is a *semigroup of fractions* of S. We remark that if a commutative semigroup is a left order in Q, then Q is commutative so that S is an order in Q. A given commutative order S may have more than one semigroup of fractions. The semigroups of fractions of S are pre-ordered by the relation $Q \geq P$ if and only if there exists an onto homomorphism $\phi : Q \to P$ which restricts to the identity on S. Such a ϕ is referred to as an S-*homomorphism*; the classes of the associated equivalence relation are the S-isomorphism classes of orders, giving us a partially ordered set $\mathcal{Q}(S)$. In the best case, $\mathcal{Q}(S)$ contains maximum and minimum elements. In a commutative order S, $\mathcal{S}(S)$ is also an order and has a maximum semigroup of fractions R, which is a Clifford semigroup. We investigate how much of the relation between $\mathcal{S}(S)$ and its semigroups of fractions can be lifted to S and its semigroups of fractions.

[*]Joint work with P. N. Ánh, V. Gould, and P. A. Grillet.

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On the coincidence of internal categories and internal groupoids

We study the difference between internal categories and internal groupoids in terms of generalised Mal'tsev properties—the weak Mal'tsev property on the one hand, and n-permutability on the other. We give conditions on internal categorical structures which detect whether the surrounding category is naturally Mal'tsev, Mal'tsev or weakly Mal'tsev. We show that these do not depend on the existence of binary products. In the second part we prove that, in a weakly Mal'tsev context, categories and groupoids coincide precisely when every relation which is reflexive and transitive is also symmetric. In varieties of algebras this latter condition is known to be equivalent to n-permutability. Moreover, in the regular context this last condition (n-permutability) is sufficient to guarantee the coincidence of groupoids and internal categories.

*This is joint work with Tim Van der Linden and part of it is also joint work with Diana Rodelo.

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Emulations of quantum Turing machines as morphisms

The category of quantum Turing machines is presented and its properties are discussed. The existence of universal quantum Turing machine, the s-m-n theorem and other results concerning (quantum) Kolmogorov complexity are analysed in the context of the proposed category. On going research work with Amílcar Sernadas and André Souto.

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Derived categories and Fourier-Mukai equivalences in algebraic geometry

Since its introduction in the 1960s by Grothendieck and Verdier, derived categories have been attracting the attention of mathematicians from various fields, in particular algebraic geometers. Mukai's breakthrough work in the 1980s showed that the notion of derived equivalence is very interesting from the geometric point of view and recently it has been playing a crescent role in birational geometry. In this talk I will survey some aspects of the theory of derived categories of sheaves and Fourier Mukai transforms in algebraic geometry. If time permits, I will also report on joint work with A. Rapagnetta and F. Viviani where we use Fourier-Mukai transforms to show that certain algebraic varieties that show up in the classical limit of the Geometric Langlands Conjecture are autodual.

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Characteristic subobjects in semi-abelian categories

We extend to semi-abelian categories the notion of characteristic subobject, which is widely used in group theory and in the theory of Lie algebras. Moreover, we show that many of the classical properties of characteristic subgroups of a group hold in the general semi-abelian context, or in stronger ones.

[*]Joint work with Alan S. Cigoli.

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Extending semilattices to frames

Each (bounded) meet-semilattice S is well known to be freely extended to its downset frame $\mathfrak{D}S$. This extension, of course, does not respect the possible joins, and the question naturally arises when and how one can extend the semilattice to a frame preserving a given part of the existing join structure. Using the Johnstone's technique of coverages and sites, and a deep injectivity result by Bruns and Lakser one can show that the range of frame extensions of S is a sub-coframe (indeed an interval) of the coframe of the sublocales of $\mathfrak{D}S$, with the injective envelope of S as the bottom.

We will also briefly mention the relation of the extensions and the Dedekind-MacNeille completion, and a few further aspects of the construction involved.

[*]Joint work with R.N. Ball.

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Étale groupoids and their quantales: functoriality

Both the C*-algebras of locally compact groupoids and the quantales of open localic groupoids are algebraic structures of "convolution type". Such constructions are not immediately functorial. For instance, on one extreme, the functor C from the category of compact Hausdorff spaces to the category of C*-algebras is contravariant, as is the open sets functor Ω from topological spaces to frames, whereas, on the other extreme, there is a covariant functor from discrete groups to C*-algebras, and, similarly, there is a covariant functor from discrete groups to quantales. In order to make sense of these extremes within a single definition one should use bicategories. This idea has appeared in several ways in the context of C*-algebras (see, e.g., [1, 2, 3]), and in this talk I explain how it applies to localic étale groupoids and their quantales. Indeed, here the situation is more satisfactory because one obtains an equivalence of bicategories, namely between the bicategory of localic étale groupoids with bi-actions as 1-cells, and the bicategory of inverse quantal frames, whose 1-cells are quantale bimodules. This is a natural functorial extension of the objects-only correspondence of [4].

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A remark on pullbacks in Gumm categories

We study some properties of pullbacks in the context of Gumm categories (see [2, 1]), which extend some known ones in the context of Mal'tsev categories. We mainly consider the cases when the base category is also regular, or almost-exact in the sense of [4]. As an application to Categorical Galois Theory, we obtain a new and simple proof of the fact that every central extension is normal in the Barr-exact Goursat context [3].

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A categorical road to decidability

It is well known that quantifier elimination plays an important role in proving decidability of a first-order theory using either proof-theoretic or model-theoretic techniques. After providing an overview of the relevant results, a one-step construction is proposed for proving quantifier-elimination adopting a model-theoretic standpoint. Several illustrations are provided.

[*]Joint work with João Rasga and Amílcar Sernadas.

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Closure operators and their duals

Categorical closure operators have been studied for almost three decades now (see in particular the monographs by Dikranjan-Tholen and Castellini on this subject) and may be regarded as an essential tool not only in categorical topology and sheaf- and topos theory, but also in algebra, order and domain theory. What is the dual notion of closure operator? When dualized from a merely order-theoretic perspective, one arrives at Vorster's notion of interior operator (S. J. Vorster, Quaestiones Mathematicae 23 (2000) 405-416) which has found renewed interest in recent papers by Castellini, Holgate and Slapal. However, when the relevant subobject lattices are complemented, all interior operators are induced by closure operators, so that truly novel applications of the notions are to be found only beyond the realm of set-based topological categories. In this talk we work with a general definition of closure operator which lends itself easily to categorical dualization. We present the basics of the theory of dual closure operators and give a variety of examples from algebra and topolgy.

[*]Joint work with Dikran Dikranjan.

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A Galois theory of monoids

We show that the adjunction between monoids and groups obtained via the *Grothendieck group* construction is admissible, relatively to surjective homomorphisms, in the sense of categorical Galois theory [3]. The central extensions with respect to this Galois structure turn out to be the so-called *special homogeneous surjections* [1, 2]. As a consequence, special homogeneous surjections are reflective amongst surjective monoid morphisms.

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*Joint work with Andrea Montoli and Diana Rodelo.

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Abstract characterisation of varieties and quasivarieties of ordered algebras

Classes of algebras whose carrier is a poset and whose operations are monotone functions and which are definable by inequalities (or, by implications using inequalities) were characterised in Birkhoff's HSP-style (or, in SP-style) by Stephen Bloom [1]. The H, S and P closure operators are related to a factorisation system of monotone surjections and order-reflecting embeddings in the category Pos of posets and monotone maps.

We show that the above factorisation system makes the category Pos *exact in 2-dimensional sense* and, as a consequence, one can characterise varieties and quasivarieties of ordered algebras as abstract categories in a similar way as for the classical case.

Namely, we prove the following ([2]):

1. A category is equivalent to a variety of ordered algebras iff it is exact in the 2-dimensional sense, has coinserters, and possesses a "nice" generator.

2. A category is equivalent to a quasivariety of ordered algebras iff it is regular in the 2-dimensional sense, has coinserters, and possesses a "nice" generator.

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*Joint work with Alexander Kurz.

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A characterisation of R_1-spaces via approximate Mal'tsev operations

For an object X in a category $\mathbb{C}$, a morphism $\mu : X^3 \to A$ is an approximate Mal'tsev operation with approximation $\alpha : X \to A$ [2] if for any object C of $\mathbb{C}$ and for any two morphisms $x, y : C \to X$ we have $\mu(x, y, y) = \alpha x = \mu(y, y, x)$. It was shown in [2] that a regular category [1] with coproducts is a Mal'tsev category [3] if and only if every object admits an approximate Mal'tsev co-operation whose approximation is a regular epimorphism. The dual of the category of topological spaces is regular, but not Mal'tsev, since not all topological spaces admit an approximate Mal'tsev operation whose approximation is a regular monomorphism (i.e. an embedding). In this talk we characterise those topological spaces which do. These spaces turn out to be precisely the R_1-spaces [4] (also known in the literature as pre-regular spaces).

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