Coproducts of Monads

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The formation of coproducts in the category of monads is important both in General Algebra (unification of algebraic theories) and in Computer Science (combination of computation effects). Special cases have been studied by various authors, e.g. coproducts with a free monad by Plotkin, Hyland and Power, and coproducts of ideal monads by Ghani and Uustalu.

We introduce the concept of separated monads: these are monads for which the underlying functor has, when restricted to monomorphisms, a complement of the monad unit. Example: over *Set* all monads are separated except for the trivial subterminal monads (sending all nonempty sets to 1). For separated monads a coproduct formula is proved, assuming certain fixed points of the underlying functors exist. In case of monads on *Set* that sufficient condition is also proved to be necessary. Corollary: a monad $\mathcal{S}$ on *Set* has coproducts with all monads iff it has a coproduct with the power-set monad. And this holds, besides for the subterminal monads, precisely for the exception monads $\mathcal{S}(X) = X + E$ (E a set of exceptions).

This is joint work with N. Bowler, P. Levy and S. Milius.