A push forward construction and the comprehensive factorization for internal crossed modules I

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Given a short exact sequence $A \to B \to C$ in an abelian category, any morphism $c\colon C' \to C$ produces by pullback a new short exact sequence with the same kernel A. Dually, any morphism $a\colon A \to A'$ produces by pushout a new short exact sequence with the same cokernel C. If the base category is semi-abelian, the first construction still produces a short exact sequence, but this is no longer true for the second one, because the pushout of a normal monomorphism is not in general a normal monomorphism.

This problem can be fixed by giving some supplementary conditions, and a *push forward* construction, that in the abelian case specializes to a pushout.

More generally, it is possible to push forward along a map (with suitable hypothesis) not only a normal monomorphisms, but any internal (pre)crossed module [3], obtaining this way a crossed module with the same cokernel. This is well known in the category of groups (see [4], for instance) and it has been recently investigated in the semi-abelian case in [2] by means of cross effects.

In this talk I will present other necessary and sufficient conditions, expressed in terms of internal actions, for the push forward construction. When the category is moreover action accessible [1], hypothesis reduce to a simplified version involving a "Peiffer-style" condition.

References

[1] D. Bourn and G. Janelidze, *Centralizers in action accessible categories*, Cahiers de topologie et géométrie différentielle catégoriques 50 n.3 (2009) 211–232.

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[3] G. Janelidze, *Internal crossed modules*, Georgian Mathematical Journal 10 n. 1 (2003) 99–114.

[4] B. Noohi, *On weak maps between 2-groups*, arXiv:math/0506313v3.

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