On the enriched Vietoris monad

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In 1922, L. Vietoris [3] defined a topology on the set of closed subsets ("Bereiche zweiter Ordnung") of a (compact Hausdorff) topological space which constitutes a topological generalisation of the "Pompeiu-Hausdorff" metric on the closed subsets of a metric space introduced a couple of years earlier by D. Pompeiu and F. Hausdorff. Later on, many authors studied also variations of this topology, such as upper/lower Vietoris topology, Fell topology, to name a few. It turns out that the upper Vietoris space is part of a monad on Top which can be restricted to the full subcategory of locally compact spaces and the (non-full) subcategory of stably compact spaces and spectral maps, as well as to compact Hausdorff spaces (now taking the Vietoris space instead). Furthermore, O. Wyler [4] showed that the algebras for the Vietoris monad on compact Hausdorff spaces are precisely the continuous lattices. Seemingly unrelated, we also mention here that A. Pultr and J. Sichler [2] characterised those spectral spaces (called f-spaces) which correspond to frames under Stone (resp. Priestley) duality for distributive lattices.

In this talk we continue thinking of topological (and other kinds of) spaces as generalised enriched categories (see [1], for instance). Consequently, when studying spaces, we will talk about weighted (co)limit in spaces, distributor, representable category, dual category, (contra/co)variant presheaf, cocompleteness and its dual(?) concept of completeness, distributivity and its dual(??), and so on. In particular, we will show that these generic categorical notions and results can be indeed connected to more "classical" topology: for topological spaces,

- the covariant presheaf monad becomes the upper Vietoris monad,
- the statement "X is totally cocomplete iff X^{op} is totally complete" specialises to O. Wylers characterisation of the algebras of the Vietoris monad on compact Hausdorff spaces, and
- the fact that distributivity does not behave well with respect to dualisation relates to the f-spaces of A. Pultr and J. Sichler.

References

[1] D. Hofmann, *Topological theories and closed objects*, Adv. Math. **215**(2) (2007), 789–824.

[2] A. Pultr and J. Sichler, *Frames in Priestley's duality*, Cah. Topol. Géom. Différ. Catég. **29**(3) (1988), 193–202.

[3] L. Vietoris, *Bereiche zweiter Ordnung*, Monatsh. Math. Phys. **32**(1) (1922), 258–280.

[4] O. Wyler, *Algebraic theories of continuous lattices*, Continuous lattices, Proc. Conf., Bremen 1979, Lect. Notes Math. 871 (1981), 390-413.