

# Neighbourhood relations in point-set topology

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Given a topological space  $B$  we consider the relation, say  $E$ , from  $P(B)$  to  $B$ , defined by:  $(U,x)$  is in  $E$  if and only if  $U$  is a neighbourhood of  $x$  in  $B$ . This is the intuition behind the notion of “neighbourhood relation” which we introduce as a span  $(N: E \rightarrow P(B), p: E \rightarrow B)$  satisfying some suitable axioms. With the appropriate morphisms it forms a category that is isomorphic to  $\text{Top}$ , the category of topological spaces. We also consider the notion of “basic neighbourhood relation” and give some examples. It turns out that this equivalent definition of topological space can be internalized, in a nice way, in a category with finite limits and strong universal relations, in particular in any topos.