Subobject and quotient lattices in locally finitely presentable categories

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We discuss the problem to what extent subobjects and quotients respectively of objects K in a locally finitely presentable category form an algebraic lattice. Our results, which essentially are (simple) consequences of well known closure properties of the class of locally finitely presentable categories, in particular lead to a completely categorical explanation of the well known facts that the subobject and congruence lattices of algebras in finitary varieties are algebraic. We also obtain new natural examples as, e.g., (1) for every object K in a locally finitely presentable category the internal equivalence relations on K form an algebraic lattice; and (2) for every (not necessarily finitary) polynomial set-functor F, the subcoalgebras of an F-coalgebra form an algebraic lattice; the same holds for the lattices of regular congruences and quotients of these F-coalgebras.

References

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