

Probabilistic metric spaces as enriched categories

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Lawvere’s ground-breaking paper [2] presenting generalised metric spaces as enriched categories has motivated much work on the reconciliation of order, metric and category theory, we refer in particular to the work of Flagg *et al.* on continuity spaces and the work of the Amsterdam research group at CWI. One amazing insight of [2] is a characterisation of the notion of Cauchy completeness for metric spaces using adjoint distributors, giving further evidence to MacLane’s motto “adjoints occur almost everywhere”. This result was further generalised in [1] to categories enriched in a *value quantale*: for such categories, Cauchy completeness can be equivalently described via distributors and via Cauchy nets. Using the conceptual power of adjunction, in this talk we show that many results linking adjoint distributors and Cauchy sequences (resp. nets) are valid under milder assumptions. We interpret our results in probabilistic metric spaces [3] seen as categories enriched in the quantale Δ of distribution functions, and show that in many cases categorical and classical notions coincide. Furthermore, we establish sufficient conditions for functors induced by morphisms of quantales to preserve Cauchy completeness, paying again special attention to the case of probabilistic metric spaces.

REFERENCES

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