The algebra and geometry of networks

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Networks of components have a compositional description in terms of the algebra of symmetric or braided monoidal categories in which each object has a commutative Frobenius algebra structure compatible with the tensor product. They also have a geometric description [8] - the free such algebra (the symmetric case) is the category of cospans of multigraphs, arrows of which have a pictorial representation. These results are in the line introduced by Joyal and Street [2], and were obtained by Sabadini and Walters with collaborators Katis and Rosebrugh in earlier work especially [4, 5, 6, 8, 9, 11], beginning with the work on relations with Carboni [1] in 1987.

The present work presents two developments.

The free braided algebra of the type described above was introduced by Rosebrugh, Sabadini and Walters [11] under the name Tangled Circuits since the geometry captures not only the connection between components but their entanglement. Determining whether two arrows are equal in this category is difficult since it seems to include the problem of classifying knots as a special case. We present some initial work in classifying a special class of arrows which we call blocked braids, that is, an arrow of the form $S \circ B \circ R$ Where $R : I \to X^n$ (X^n the tensor power of X), $B : X^n \to X^n$ is a braid on n strings, and $S : X^n \to I$. Dirac's belt trick, the unwinding of two full twists of a belt, is an example of an equation in this category. We show that blocked braids on less than four strings are finite in number whereas with four or more strings the number is infinite.

The second development is a graphic tool for calculating compositionally cospans of multigraphs and for searching the state space of sequential or parallel networks in which the components have state. The state space of a sequential network is a colimit, and of a parallel network is a limit [8]. This has close connections with our paper [3] and recent work of Sobocinski [12, 13] on Petri nets.

References

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