The nerve of an algebraic 2-type

Ivan Yudin

For the sake of this talk, an algebraic 2-type is a triple (G, A, f), where G is a group, A is an abelian group, and $f : G^3 \to A$ is a map that satisfies cocycle condition. It is possible to define morphisms between algebraic 2-types in such a way that the resulting isomorphisms classes are in one-to-one correspondence with the triples (G, A, ϕ), where G is a group, A is an abelian group and $\phi \in H^3(G, A)$. It is known that these triples are also in one-to-one correspondence with homotopy equivalence classes of topological 2-types. The construction of a topological 2-type corresponding to a given triple (G, A, ϕ) usually goes through a construction of a free crossed module corresponding to (G, A, ϕ) and then taking its nerve. The drawback of this approach is that even for triples (G, A, ϕ) with finite G and A, we get simplicial set with infinitely many cells in every dimension.

In my talk, I will describe a functor N from the category of algebraic 2-types to the category of 2-types in the category of simplicial sets such that

1) the homotopy equivalence class of $N(G, A, f)$ corresponds to $(G, A, [f])$;

2) the set of k-cells of $N(G, A, f)$ is isomorphic to $G^k \times A^{k(k-1)/2}$.