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The pointfree Daniell Integral

A satisfactory theory of abstract integration, i.e., the study of positive continuous linear functionals on archimedean vector lattices, requires two things. The first is agreement on a unit of measurement; a weak unit is more than is necessary but a truncation is well suited to the task. The second is a satisfactory notion of pointwise convergence on the underlying vector lattice. Surprising as it may seem, the most satisfactory handle on pointwise convergence emerges from pointfree topology. The classical Daniell approach to integration is to start with an integral on a small family of "simple" functions, and then extend it to a larger family of functions using pointwise convergence. Although it requires transfinite iteration, this can be done just as well in the much more general context of pointfree pointwise convergence. The resulting extension is maximal, both with respect to having the smaller family pointwise dense in the larger, and with respect to having the smaller family epically embedded (in the category of truncated archimedean l-groups) in the larger.