

*Canonical extensions as a form of pointfree topology*

In pointfree topology *frames*, meant to model the opens of a space, replace the traditional concept of a topological space. However, an important part of topology is precisely the tension between the continuous (carried by the frame of opens) and the discrete (carried by the points). In the canonical extension of Jónsson-Tarski, Boolean spaces are replaced by the embedding of the corresponding Boolean algebra into a complete Boolean algebra, which is equivalent via discrete duality to the set of points of the space, thus accounting both for the opens and the points of the space. In joint work with Jónsson, we generalized this concept to bounded distributive lattices, and then with Harding, to arbitrary bounded lattices. In the work with Harding we gave an alternate characterization of the canonical extension and showed that it is available in a constructive setting. This view of canonical as an alternative form of pointfree topology raises a number of questions and puts the study of Alexandrov spaces and the discrete duality with posets in a more central position. In this context we revisit joint work with Marcel Ern  and Aleř Pultr on the study of the relationship between complete congruences and subspaces.