

On the lattice of z -ideals of a commutative ring

Let A be a commutative ring with identity. An ideal I of A is called a z -ideal if whenever two elements of A belong to the same maximal ideals and one of the elements is in I , then so is the other. We prove that the lattice of z -ideals of a commutative ring with identity is a coherent frame. A ring with zero Jacobson radical is shown to be feebly Baer precisely when its frame of z -ideals is feebly projectable. Denote by $Z\text{Id}(A)$ the frame of z -ideals of a ring A . We show that the assignment $A \mapsto Z\text{Id}(A)$ is the object part of a functor $\mathbf{CRng}_3 \rightarrow \mathbf{CohFrm}$, where \mathbf{CRng}_3 designates the category whose objects are commutative rings with identity and whose morphisms are the ring homomorphisms that contract z -ideals to z -ideals.