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The formal theory of relative monads

The notion of *relative monad*, first introduced in the seminal paper [1], arose to resolve size-issues related to monad-like correspondences; for example, the presheaf construction $A \mapsto [A^o, \mathbf{Set}]$ fails to be a pseudomonad for size reasons, since it sends *small* categories to *locally small* ones, making it impossible to define a multiplication. These gadgets can be neatly characterized as monoids internal to the *skew* monoidal category [5] of functors $[C, D]$. A rather satisfying theory of monads can be regained in the relative setting if we allow T to be a pseudofunctor (this is precisely the case of the presheaf construction): [3] proves that the Kleisli bicategory of $P = [-, \mathbf{Set}]$ coincides with the bicategory $\mathbf{Prof}$ of Set-enriched profunctors.

In the present talk we suggest that these phenomena all fit in a bigger framework: relative monads exist in every 2-category $\mathcal{K}$ (in the same way 2-monads do); the *horizontal Kleisli bicategory* [2] of a relative monad $T : \mathbb{A} \to \mathbb{B}$ between virtual double categories becomes a *virtual equipment* (*ibi*) as soon as $T = [-, \Omega]$ is induced by a dualizing object of $\mathcal{K}$, and an equipment [6, 7] if Ω satisfies some additional properties. When T is the lax idempotent, relative pseudomonad that arises from the presheaf construction of an abstract Yoneda structure [4], this suggests how the relative monad itself connects Yoneda structures and equipments, complementary frameworks in which to deploy formal category theory.

References

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