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Sobriety and congruence biframes

It is well known that the lattice of congruences on a frame is itself a frame. Furthermore, such a congruence frame naturally has the structure of a *strictly zero-dimensional biframe* [1]. The Skula topology on a T_0 space provides another class of strictly zero-dimensional biframes, allowing for the pointfree representation of even non-sober T_0 spaces. In this way, strictly zero-dimensional biframes provide a setting where we may meaningfully discuss sobrification in pointfree topology.

We will see that congruence biframes play a role analogous to sober spaces in this setting. We will describe a few results about congruence biframes that mirror classical results about sober spaces and sobrification. These include the characterisation of congruence biframes by the existence of smallest dense sublocales [2] and a characterisation in terms of bicompleteness with respect to a quasi-uniformity as in [3].

References

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[2] G. Manuell, Strictly zero-dimensional biframes and a characterisation of congruence frames, *Appl. Categ. Structures* 26(4) (2018) 645–655.

[3] H-P.A. Künzi and N. Ferrario, Bicompleteness of the fine quasi-uniformity, *Math. Proc. Camb. Phil. Soc.* 109(1) (1991) 167–186.